Internet Appendix for

Tackling the Volatility Paradox: Spillover Persistence and Systemic Risk

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A Data and Summary Statistics

A.1 Variable Definitions

Table IA.1: Variable Definitions and Data Sources.

Equity market data is at daily frequency, all other variables are at annual frequency. All systemic risk measures and firm and bank characteristics are winsorized at 1%/99%.

Variable	**Definition**
Equity Market Data	
STOCK_PRICE	Daily unadjusted and unpadded price of common equity. *Source*: Thomson Reuters Datastream
OUTSTANDING_SHARES	Daily number of outstanding shares of common equity. *Source*: Thomson Reuters Datastream
MARKET_VALUE	Daily market value of equity in USD. *Source*: Thomson Reuters Datastream
(Systemic) Risk Measures	
ΔCoSP(τ)	Likelihood of losses of the system τ days after losses of the institution in excess of the reference level $q = 0.05$
AVERAGE_ΔCoSP ($\bar{\psi}$)	Average level of ΔCoSP across time-lags
SPILLOVER_PERSISTENCE ($\bar{\tau}$)	Systemic-risk–weighted average time-lag
ΔCoVaR	Change in the system's Value-at-Risk conditional on a firm being under distress compared to its median state
MES	Firm's average equity return loss conditional on large system losses on the same day
Macroeconomic Characteristics	
NFCI	Federal Reserve Bank of Chicago's National Financial Conditions Index; annual average. *Source*: FRED
INFLATION	Δlog(Consumer Price Index); annual rate, country-level. *Source*: BIS
GDP_GROWTH	Δlog(real GDP); annual rate, country-level. *Source*: OECD
INVESTMENT_GROWTH	Δlog(investment/GDP); annual rate, country-level. *Source*: OECD
CREDIT_GROWTH	Δlog(credit/GDP); annual rate, country-level. *Source*: BIS
CRISIS	Indicator for the occurrence of banking crises. *Source*: Laeven and Valencia (2020)
OUTPUT_LOSS	3-year cumulative deviation from GDP trend associated with banking crises. *Source*: Laeven and Valencia (2020)

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Table IA.1 – *Continued from previous page*

Variable	**Definition**
log(INTEREST_RATE)	*log*(10-year government bond rate); annual average of weekly rate, continent-level. *Source*: see Table IA.2
3M_YIELD_CHANGE	Weekly change in 3-month government bond rates; annual average. *Source*: see Table IA.2
TERM_SPREAD_CHANGE	Weekly change in yield spread between 10-year and 3-month government bond rates; annual average. *Source*: see Table IA.2
TED_SPREAD	Spread between 3-month Libor (interbank) and 3-month government bond rates; annual average. *Source*: see Table IA.2
CREDIT_SPREAD_CHANGE	Weekly change in the spread between Moody's Baa rated bonds and 10-year government bond rates; annual average. *Source*: see Table IA.2
MARKET_RETURN	Weekly market return of system-specific MSCI indices; annual average. *Source*: see Table IA.2
EQUITY_VOLATILITY	22-day rolling window market return of system-specific MSCI indices; annual average. *Source*: see Table IA.2
BOOM	Indicator for whether a country experiences a stock market boom. *Source:* Brunnermeier et al. (2020)
BUST	Indicator for whether a country experiences a stock market bust. *Source:* Brunnermeier et al. (2020)
BOOM_LENGTH	Current length of a country's stock market boom. *Source:* Brunnermeier et al. (2020)
BUST_LENGTH	Current length of a country's stock market bust. *Source:* Brunnermeier et al. (2020)
BURST_DISTANCE	Current distance to a country's stock market bubble's burst. *Source:* Own calculation based on data from Brunnermeier et al. (2020)
Firm Characteristics (*Source: Worldscope.*)	
SIZE	log(total assets)
LEVERAGE	Total assets / market value of common equity
MARKET_TO_BOOK	Market value of equity / book value of equity
Bank Characteristics (Ban & Bro Sample) (*Source: BankFocus if not stated otherwise*)	
SIZE	log(total assets)
LEVERAGE	Total assets / market value of equity *Source*: BankFocus (total assets) and Worldscope (market value)
DEMAND_DEPOSITS	Customer deposits that can be withdrawn immediately without notice or penalty / total assets
INTANGIBLE_ASSETS	(Goodwill + other intangible assets) / total assets

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Table IA.1 – *Continued from previous page*

Variable	**Definition**
IMPAIRED_LOANS	Impaired & non-performing exposure on customer and inter-bank loans before loan loss reserves / total assets
LIQUIDITY_RATIO	Liquid assets (cash and balances with central banks, net loans & advances to banks, reverse repos, securities borrowed & cash collateral, and financial assets: trading and at fair value through P&L less any mandatory reserve deposits with central banks) / deposits and short-term funding
CDS	Total credit default swap notional / total assets
Fire Sale Sample	
EXPOSED	Indicator whether insurer's total P&C premiums written in Alabama, Louisiana, and Mississippi (at insurance group level) from 2004Q3 to 2005Q2 are in the upper quartile of the distribution across US insurers. *Source:* own calculation based on insurers' quarterly Schedule T filings to the NAIC retrieved from S&P Global Market Intelligence
POST_KATRINA	Indicator for August 25, 2005, and onwards

Table IA.2. Region-level macroeconomic state variables and data sources.

The table depicts the region-level macroeconomic variables, which also serve as state variables to estimate ΔCoVaR with quantile regressions, and compares them to the state variables used by Adrian and Brunnermeier (2016) for the U.S. The choice of state variables is motivated by that in Brunnermeier et al. (2020).

Used by AB2016	**Data used instead**					
	North America	**Europe**	**Japan**	**Australia**	**Asia (ex Japan)**	**Africa**
10Y treasury rate	US 10Y treasury rate (FRED)	German 10Y govt. bond rate (Datastream)	Japanese 10Y govt. bond rate (Datastream)	Australian 10Y govt. bond rate (Datastream)	Indian 10Y govt. bond rate (Datastream)	South African 10Y govt. bond rate (Datastream)
3M T-Bill rate	US 3M T-Bill rate (FRED)	German 3M govt. bond rate (Datastream)	Japanese 3M govt. bond rate (Datastream)	Australian 3M govt. bond rate (Datastream)	Indian 3M govt. bond rate (Datastream)	South African 3M govt. bond rate (Datastream)
3M Libor rate	3M Libor rate (FRED)	3M Fibor rate (Datastream)	3M Japanese Libor rate (FRED)	Australian 3M interbank rate (Datastream)	Indian 91-day T-bill rate (Datastream)	South African 3M interbank rate (Datastream)
Moody's Baa rated bonds	Moody's Baa rated bonds (FRED)	Moody's Baa rated bonds (FRED)	Moody's Baa rated bonds (FRED)	Moody's Baa rated bonds (FRED)	Moody's Baa rated bonds (FRED)	Moody's Baa rated bonds (FRED)
S&P500	MSCI North America (Datastream)	MSCI Europe (Datastream)	MSCI Japan (Datastream)	MSCI Australia (Datastream)	MSCI Asia (excl Japan) (Datastream)	MSCI Africa (Datastream)
CRSP equity market index	MSCI North America (Datastream)	MSCI Europe (Datastream)	MSCI Japan (Datastream)	MSCI Australia (Datastream)	MSCI Asia (excl Japan) (Datastream)	MSCI Africa (Datastream)

A.2 Variable Construction

A.2.1 Macroeconomic Characteristics. In many analyses, I control for macroeconomic variables that capture key differences in economic environments, namely inflation, GDP growth, credit growth, investment growth, and an indicator for banking crises (all at country-level), and the logarithm of the annual average of the 10-year government bond rate (at region level).[1]

Additionally, I use granular variables on funding conditions and financial markets (motivated by their use by Adrian and Brunnermeier, 2016), namely annual averages of the weekly changes in 3-month government bond rate, weekly changes in the slope of the yield curve (10-year and 3-month government bond rate spread), the TED spread (3-month interbank and government bond rate spread), weekly changes in credit spreads (between Moody's Baa-rated bonds and the 10-year government bond rate), and the weekly equity market return and volatility. I use different government bond rates, interbank market rates, and equity market indices for different geographical regions (Europe, North America, Asia, Japan, and Australia). I retrieve all available data on a daily basis, interpolate missing data by using cubic spline interpolation, and winsorize each variable at 1% and 99%. The data sources are St. Louis FRED database and Thomson Reuters Datastream. A detailed description of variable definitions and data sources is given in Tables IA.1 and IA.2.

A.2.2 Firm Characteristics. I consider several firm-level variables that have been shown to be relevant for systemic risk, namely firm size (the logarithm of total assets), the ratio of market to book value, and leverage (the ratio of total assets to the market value of equity). Annual data for these variables are from Thomson Reuters Worldscope.

Additionally, I zoom in on granular characteristics of banks and broker-dealers. For this purpose, I retrieve detailed bank-level data from 1990 to 2016 for all banks featured in both Moody's Analytics BankFocus and the sample of systemic risk measures. I consider bank-level variables that provide granular information on banks' liquidity profile, namely the relative size of intangible assets, demand deposits, time deposits, loans, and impaired (and non-performing) loans (all scaled by total assets), and banks' liquidity ratio defined by liquid assets over deposits and short-term funding.[2] For additional analyses on bank risk-taking, I also retrieve data on banks' CDS exposure, which is the CDS notional as a share of total assets. To ensure consistency in accounting, I use total assets from BankFocus as a scaling

[1]The annual average of the 10-year government bond rate is strictly positive throughout the whole sample after merging with systemic risk measures. I use its logarithm following Brunnermeier et al. (2020). The results are robust to using the actual level of the interest rate level instead of its logarithm.

[2]Detailed variable definitions are given in Table IA.1. If available, I use banks' consolidated balance sheet, and the unconsolidated balance sheet otherwise.

factor for all bank-related variables and also re-calculate size and leverage for banks using BankFocus in all regressions for the sample of BankFocus firms.

A.2.3 Exposure to Hurricane Katrina. For each US insurer, I calculate the share of total P&C insurance premiums written (at the group level) in Alabama, Louisiana, and Mississippi relative to total premiums written in the year prior to Katrina (i.e., in quarters 2004Q3 to 2005Q2). US insurers in the upper quartile of the cross-sectional distribution of premium shares are defined as exposed to Katrina, remaining US insurers are in the control group.[3]

US insurance companies report premiums for direct insurance business (excluding reinsurance business) at the state-level in Schedule T of their quarterly statutory filings. I retrieve this data from S&P Global Market Intelligence. To detect reporting errors, I compare the sum of premiums across states reported on Schedule T with that reported in the insurer's overview filings and exclude insurer-quarters if there is a discrepancy larger than 50 thd USD and 50% of the average total direct premiums reported across the filing pages. I then calculate (1) the sum of total P&C premiums written in Louisiana, Mississippi, and Alabama and (2) the sum of total direct premiums written from 2004Q3 to 2005Q2 at the insurance group - state level.

To merge premiums to equity market data, I retrieve insurer groups' stock tickers and CUSIP identifiers from S&P Global Market Intelligence and match these to CUSIPs and stock tickers, and manually check the resulting matching. In the sample of all (51) matched insurance groups, I flag insurers as exposed to hurricane Katrina if they are headquartered in the US and the ratio of premiums written in exposed states is in the upper quartile of the cross-sectional distribution, and all other insurers as unexposed. By accounting for headquarter location, I assign two non-US insurers to the control group which would otherwise be treated (AXA and Beazley). The reason is that US premiums written are only a small fraction of the premiums written by these insurers.[4]

A.2.4 Bubbles. Bubble indicators are based on the well-established Backward Sup Augmented Dickey-Fuller (BSADF) approach by Phillips et al. (2015a,b) and Phillips and Shi

[3] Since life insurers were relatively unaffected by the hurricane, it is reasonable to include them in the control group. Although many lives were lost during Katrina, most of them were uninsured (see Towers Watson, "Hurricane Katrina: Analysis of the Impact on the Insurance Industry" available at `https://biotech.law.lsu.edu/blog/impact-of-hurricane-katrina-on-the-insurance-industry-towers-watson.pdf`).

[4] In 2005, less than 7% of AXA's P&C gross premiums were written in the US (see Annual Report 2005). In 2009, 10% of Beazley's gross premiums were written in the US (*Source: S&P Global Market Intelligence*).

(2018), applied to the main stock price indices in 17 countries from 1987 to 2015.[5] Bubble characteristics include the current length of a boom or bust. Bubble indicators are merged to the baseline sample of systemic risk measures and firm characteristics at the firm-year level.[6] The "bubbles sample" covers 33 bubbles, 17 countries, and 693 financial firms from 1989 to 2015.[7]

[5]The BSADF approach uses multiple Augmented Dickey-Fuller tests to identify non-stationary behavior in asset prices. For methodological details I refer to Brunnermeier et al. (2020), who kindly shared their sample of bubble indicators with me.

[6]I label a firm-year as stock market boom or bust observation if the respective bubble phase is present in at least 6 months of the firm's headquarter country in that year.

[7]The sample includes Australia, Belgium, Canada, Denmark, Finland, France, Germany, Great Britain, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United States.

A.3 Additional Summary Statistics

Table IA.3. Additional summary statistics.
Boom & bust length & years summary statistics are provided conditional on bubble occurrence. Variable descriptions and data sources are provided in Table IA.1.

Fragility sample						
SIZE_{t-1}	8,476	2.61	2.61	2.30	-1.10	6.52
LEVERAGE_{t-1}	8,476	11.33	6.02	15.54	0.79	40.12
$\text{MARKET-TO-BOOK}_{t-1}$	8,476	1.69	1.28	1.47	0.47	4.24
Ban & Bro sample						
SIZE_{t-1}	1,686	3.91	3.74	1.73	1.13	7.14
LEVERAGE_{t-1}	1,686	14.58	9.74	14.67	2.96	42.25
$\text{MARKET-TO-BOOK}_{t-1}$	1,686	1.46	1.27	0.90	0.45	3.04
$\text{LIQUIDITY\_RATIO}_{t-1}$	1,686	0.44	0.30	0.64	0.05	1.03
$\text{DEMAND\_DEPOSITS}_{t-1}$	1,686	0.20	0.17	0.15	0.02	0.47
$\text{IMPAIRED\_LOANS}_{t-1}$	1,686	0.02	0.01	0.02	0.00	0.05
$\text{INTANGIBLE\_ASSETS}_{t-1}$	1,686	0.02	0.01	0.03	0.00	0.07
Macro controls						
INFLATION	8,476	2.04	1.98	1.54	-0.22	4.67
GDP_GROWTH	8,476	4.11	4.21	2.80	-1.65	8.31
INVESTMENT_GROWTH	8,476	-0.37	0.40	4.07	-6.91	4.22
CREDIT_GROWTH	8,476	1.23	1.07	3.68	-4.55	7.26
log(INTEREST_RATE)	8,476	0.98	1.30	0.98	-1.26	2.06
Market controls						
3M_YIELD_CHANGE	8,476	-0.52	-0.07	2.11	-3.95	2.50
TERM_SPREAD_CHANGE	8,476	0.06	-0.26	2.30	-2.48	2.93
TED_SPREAD	8,476	37.45	30.85	31.81	0.12	101.73
CREDIT_SPREAD_CHANGE	8,476	0.09	-0.08	1.88	-3.17	3.31
MARKET_RETURN	8,476	0.13	0.20	0.39	-0.66	0.61
EQUITY_VOLATILITY	8,476	1.05	0.97	0.45	0.49	2.08
Bubbles sample						
BOOM_LENGTH	1,197	2.14	1.67	1.67	0.00	4.92
BUST_LENGTH	1,197	0.34	0.00	0.57	0.00	1.33
$\text{BOOM\_YEARS}_{(t-4):t}$	1,197	2.73	3.00	1.33	1.00	5.00
$\text{BUST\_YEARS}_{(t-4):t}$	1,197	0.41	0.00	0.70	0.00	2.00

B Empirical Methodology and Estimation Details

B.1 Firm's and System's Return

A firm's and system's equity return are mechanically correlated if the system's index included the firm. This might bias systemic risk measures. I alleviate this concern by excluding firm I from the associated system S for each pair (I, S) as described in the following.

Denote by MC_t^I the market capitalization of firm I at time t in USD. By P_t^I I denote a firm I's unpadded and unadjusted price of common equity in local currency, and by N_t^I the number of shares of the firm's common equity. A system is given by a subset $S \subseteq \{1, ..., N\}$, where N is the number of all firms in the sample. Then, the index for system S excluding firm $I \in \{1, ..., N\}$ is given as the weighted average of remaining firms' returns:

$$\text{INDEX}_t^{S|I} = \text{INDEX}_{t-1}^{S|I} \sum_{s \in S \setminus \{I\}} \frac{MC_{t-1}^s}{\sum_{j \in S \setminus \{I\}} MC_{t-1}^j} \frac{P_t^s N_t^s}{P_{t-1}^s N_{t-1}^s}. \tag{IA.1}$$

The system's log equity return is

$$r_t^S = r_t^{S|I} = \log\left(\frac{\text{INDEX}_t^{S|I}}{\text{INDEX}_{t-1}^{S|I}}\right) \tag{IA.2}$$

and the firm's log equity return is

$$r_t^I = \log\left(\frac{P_t^I N_t^I}{P_{t-1}^I N_{t-1}^I}\right). \tag{IA.3}$$

B.2 Estimation Details

Denote by $D_t^I = \mathbb{1}_{\left\{-r_t^I \geq VaR^I(q)\right\}}$ and $D_t^S = \mathbb{1}_{\left\{-r_t^S \geq VaR^S(q)\right\}}$ binary random variables for large losses of financial institution I and the system S, respectively, where the stationary distribution of $(r_t^x)_t$ satisfies $\mathbb{P}(-r_t^x \geq VaR^x(q)) = q$ for $x \in \{S, I\}$. Assume that $(D_t^I, D_t^S)_t$ is a stationary time series with the time-invariant means $\mathbb{P}(D_t^I = 1) = \mathbb{P}(D_t^S = 1) = q$ and variances $\mathbb{E}[(D_t^I - q)^2] = \mathbb{E}[(D_t^S - q)^2] = q(1-q)$. Then, ΔCoSP equals

$$\Delta\text{CoSP}_\tau = (1-q) \cdot r_{CC}(\tau), \tag{IA.4}$$

where $r_{CC}(\tau)$ is the (time-invariant and normalized) cross-correlation function of $(D_t^I, D_t^S)_t$, defined as

$$r_{CC}(\tau) = \frac{\mathbb{E}\left[(D_t^I - q)(D_{t+\tau}^S - q)\right]}{q(1-q)}. \tag{IA.5}$$

Using a standard non-parametric estimator for $r_{CC}(\tau)$, a non-parametric estimator for ΔCoSP is given by

$$\Delta\widehat{\text{CoSP}}_\tau = \frac{1}{q(n-\tau)} \sum_{t=1}^{n-\tau} \mathbb{1}_{\left\{-r_t^I \geq \widehat{VaR}^I(q),\, -r_{t+\tau}^S \geq \widehat{VaR}^S(q)\right\}} - q, \tag{IA.6}$$

where the Value-at-Risk estimator is the negative of the nq-th (or $[nq]+1$)-th) order statistic of returns if nq^x is an integer (if it is not).

To compute Spillover Persistence, I assume that ΔCoSP is exponentially declining with a larger time-lag, $\Delta\text{CoSP}_\tau = \alpha e^{\beta\tau}$ with $\alpha > 0$ and $\beta < 0$. This assumption is motivated by the dynamics of the non-parametric estimate $\Delta\widehat{\text{CoSP}}_\tau$. I estimate the parameters α and β by fitting $\Delta\widehat{\text{CoSP}}_\tau$ to $ae^{\beta\tau}$ individually for each institution and estimation window using Matlab's trust-region-reflective algorithm. I disregard observations with $\alpha \leq 0$ or $\beta \geq 0$ because, in such cases, there is either no systemic risk present or the dynamics of ΔCoSP are implausible (as they would imply that tail-returns are more correlated when they are further apart).

Figure 1 depicts the non-parametric and parametric estimates of ΔCoSP for an exemplary institution. In Figure 1 (a), from a relatively tranquil market period, ΔCoSP is clearly exponentially declining. Instead, in Figure 1 (b), from crisis times, ΔCoSP is almost constant. In both cases, the parametric estimate fits the dynamics of the non-parametric estimate very well.

An important concern is that the parametric estimation of ΔCoSP induces a systematic bias. I assess that concern in Figure IA.1. I start by examining the difference between the non-parametric and parametric estimates pooled across all time-lags and firms. Figure IA.1 (a) shows that the average difference is essentially zero and its distribution symmetric around zero in all years. This result strongly suggests that there is no systematic bias resulting from the parametric estimation of ΔCoSP. The absolute value of the 10th and 90th percentile of differences is approximately $\pm 5\%$. The symmetry in the distribution is consistent with the absence of a systematic bias, whereas the levels suggest that estimation errors are contained.

The most likely cause for a potential bias in the parametric estimation is the presence of negative values of ΔCoSP. First, it is important to note that the parametric form $\alpha e^{\beta\tau}$ allows for systematically negative ΔCoSP (in this case, it is $\alpha < 0$). More generally, ΔCoSP may be negative for two reasons: because of estimation errors or because its true value is negative. Exemplary evidence is provided by Figure 1 (a), in which $\Delta\widehat{\mathrm{CoSP}}$ drops below zero only in some instances, which are clearly estimation errors around its average dynamics.

To examine the occurrence of negative values of $\Delta\widehat{\mathrm{CoSP}}$, Figure IA.1 (b) plots the share of all firm-by-year observations with at least x negative time-lags. Whereas in almost 90% of observations, there is at least one time-lag with a negative value of $\Delta\widehat{\mathrm{CoSP}}$, in only 10% of observations, half (25) of the time-lags are associated with negative values. There are considerably less instances of three consecutive time-lags with negative $\Delta\widehat{\mathrm{CoSP}}$. In only 5% of firm-year pairs, at least one fifth of the time-lags τ exhibit a negative value of $\Delta\widehat{\mathrm{CoSP}}$ and are followed by lags $j \in \{\tau+1, \tau+2\}$ with $\Delta\widehat{\mathrm{CoSP}}_j < 0$. Thus, time-lags with negative values of $\Delta\widehat{\mathrm{CoSP}}$ are typically *not* followed by lags with negative values of $\Delta\widehat{\mathrm{CoSP}}$ but, instead, occur in isolation. These results are consistent with negative values of $\Delta\widehat{\mathrm{CoSP}}$ resulting from estimation errors rather than from systematically negative ΔCoSP.

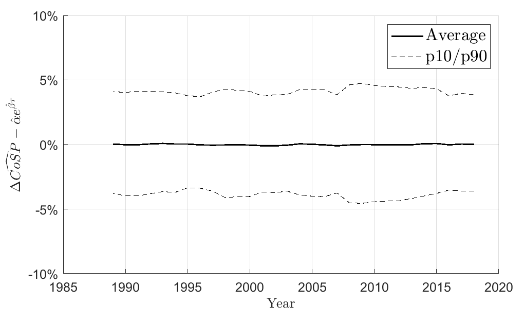
I disregard observations when the fitted parameters of $\alpha e^{\beta\tau}$ are such that $\alpha \leq 0$ or $\beta \geq 0$ or when Average ΔCoSP is below 10^{-5}. Figure IA.1 (c) shows that these criteria disregard less than 25% of observations and, in the second half of the sample, less than 15% of observations. This provides further support that the parametric estimation approach is appropriate.

Finally, in Figure IA.1 (d), I compare the baseline (parametric) measure for Spillover Persistence with an alternative (non-parametric) version that weights time-lags with $\Delta\widehat{\mathrm{CoSP}}$, allowing for negative weights $\Delta\widehat{\mathrm{CoSP}} < 0$. The figure shows substantial deviation between these two measures when Spillover Persistence is small. The large dispersion of the non-parametric estimate in these cases suggest a significant impact of estimation errors. Moreover, the non-parametric Spillover Persistence frequently drops below zero, inconsistent with its interpretation as average time-lag.

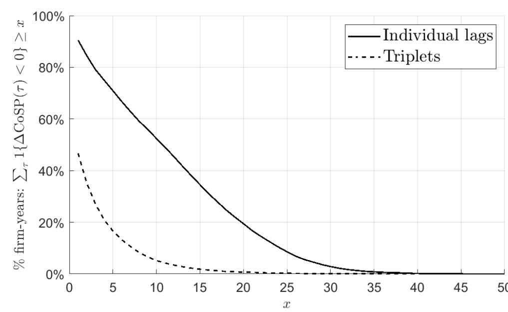
Taken together, these observations suggest that the parametric estimation procedure for ΔCoSP does not create a systematic bias and is appropriate especially in the context of estimating Spillover Persistence.

Figure IA.1. Estimation Details.

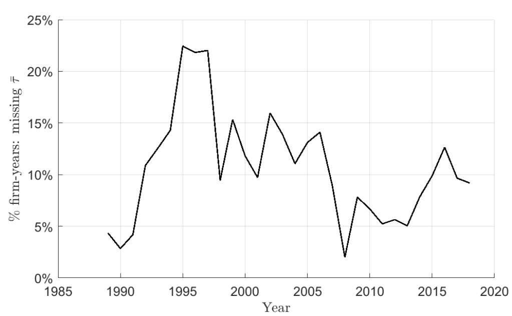
Figure (a) plots the average and 10/90th percentiles of the pooled distribution of the difference between the non-parametric and parametric estimate for ΔCoSP_τ across firms and time-lags τ. Figure (b) plots the share of firm-by-year observations with at least x individually negative time-lags (solid line), i.e., $\Delta\widehat{\text{CoSP}}_\tau < 0$ for at least x time-lags τ, and with at least x consecutively negative time-lags (dashed line), i.e., $\Delta\widehat{\text{CoSP}}_\tau < 0$ and $\Delta\widehat{\text{CoSP}}_{\tau+1} < 0$ and $\Delta\widehat{\text{CoSP}}_{\tau+2} < 0$ for at least x time-lags τ. Figure (c) plots the share of firms with a non-parametric estimate for ΔCoSP but not for Spillover Persistence. Figure (d) is a scatter plot of all observations for Spillover Persistence fitted to $\alpha e^{\beta\tau}$ against that based on the non-parametric estimate $\Delta\widehat{\text{CoSP}}$ (allowing that $\Delta\widehat{\text{CoSP}} < 0$).



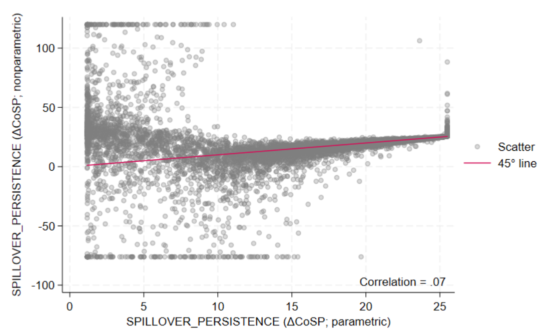
(a) Residuals from Exponential Fit.



(b) Observations with Negative ΔCoSP_τ.



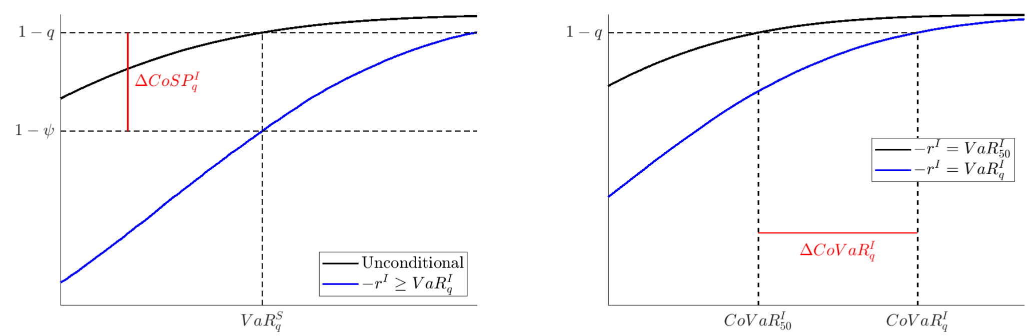
(c) Observations without Exponential Fit.



(d) Spillover Persistence: baseline vs. non-parametric.

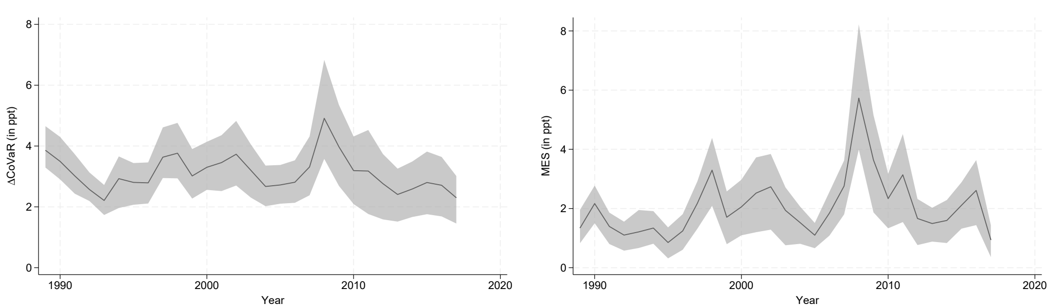
C Additional Figures and Tables

Figure IA.2. Conceptual Illustration of ΔCoSP in Comparison with ΔCoVaR.
The figures depict (conditional) cumulative distribution functions (cdfs) of the system's return losses ($-r^S$) and the quantiles and probabilities that correspond to (a) ΔCoSP and (b) ΔCoVaR. In Figure (a), the upper (black) solid line is the unconditional cdf and the lower (blue) is the cdf conditional on the institution's return losses exceeding their Value-at-Risk ($-r^I \geq VaR_q^I$). CoSP equals one minus the value of the conditional cdf at the system's Value-at-Risk. ΔCoSP is the difference between the two cdfs at the system's Value-at-Risk. In Figure (b), the upper (black) solid line is the cdf conditional on the institution's return losses being at their median and the lower (blue) is the cdf conditional on the institution's return losses being at their distressed Value-at-Risk. CoVaR is the respective quantile at $1-q$. ΔCoVaR is the difference between the two quantiles corresponding to $1-q$.



(a) ΔCoSP. (b) ΔCoVaR.

Figure IA.3. Contemporaneous Systemic Risk Measures: Evolution over Time.
The figures depict the annual mean and 25th and 75th percentiles of ΔCoVaR and MES across firms.



(a) ΔCoVaR. (b) MES.

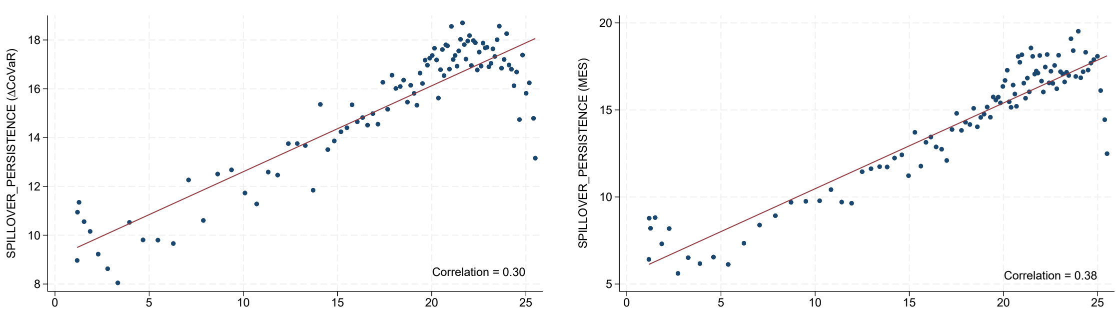
Table IA.4. Correlation of Spillover Persistence with Other Measures.

This table reports the correlation of SPILLOVER_PERSISTENCE based on ΔCoSP with other systemic risk measures and SPILLOVER_PERSISTENCE based on other systemic risk measures as well as the corresponding adjusted R^2.

	1	2	3	4	5
Measure	AVERAGE_ ΔCoSP	ΔCoVaR	MES	SPILLOVER_PERSISTENCE (ΔCoVaR)	SPILLOVER_PERSISTENCE (MES)
Correlation	0.64	0.09	0.13	0.30	0.38
Adj. R^2	0.41	0.01	0.02	0.09	0.14

Figure IA.4. Comparison of Spillover Persistence across Different Systemic Risk Measures.

These figures plot SPILLOVER_PERSISTENCE based on ΔCoSP (x-axis) against that based on (a) ΔCoVaR and (b) MES (y-axis) as binscatter plots based on firm-by-year–level observations.

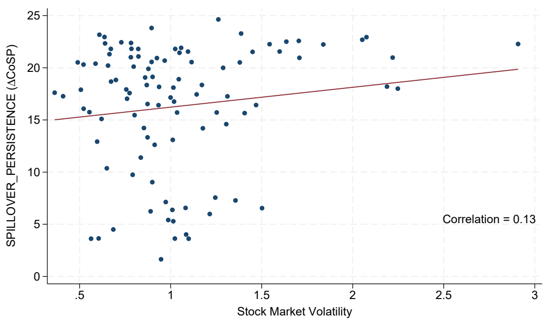


(a) ΔCoSP and ΔCoVaR.

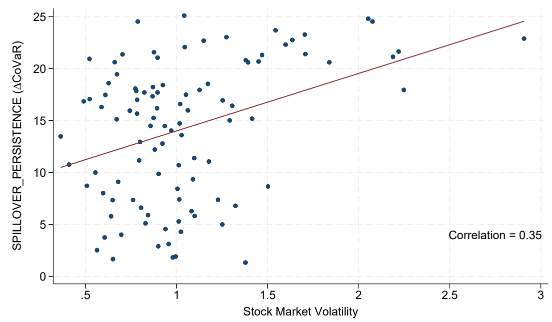
(b) ΔCoSP and MES.

Figure IA.5. Correlation of Spillover Persistence with Stock Market Volatility.

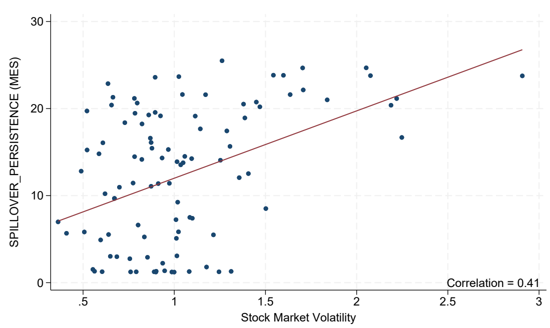
These figures plot the annual average of the 22-day trailing standard deviation of the system's equity returns (x-axis) against SPILLOVER_PERSISTENCE for the system's median institution (y-axis) based on (a) ΔCoSP, (b) ΔCoVaR, (c) MES as binscatter plots based on system-by-year–level observations.



(a) Based on ΔCoSP.



(b) Based on ΔCoVaR.



(c) Based on MES.

D Sensitivity Analyses

D.1 Robustness

Table IA.5. Robustness: Fire Sales.

Each column presents difference-in-difference estimates for the effect of hurricane Katrina on SPILLOVER_PERSISTENCE (based on ΔCoSP) of exposed US property & casualty insurers relative to other U.S. insurers:

$$\bar{\tau}_{i,t} = \text{POST-KATRINA}_t \times \text{EXPOSED}_i + u_i + \varepsilon_{i,t},$$

where u_i are firm fixed effects. POST-KATRINA equals 1 from August 25, 2005 onwards, and zero otherwise. EXPOSED equals 1 if an insurer's share of total P&C premiums in Alabama, Louisiana, and Mississippi from 2004Q3 to 2005Q2 relative to all insurance premiums is in the upper quartile across all U.S. insurers. The sample is at the firm-day level. In columns 1 and 2 it runs from August 11 to September 12, 2005, and in columns 3 and 4 from August 18 to September 5, 2005. In columns 1 and 2, SPILLOVER_PERSISTENCE is based on ΔCoSP, in column 3 based on ΔCoVaR, and in column 4 based on MES. t-statistics are shown in brackets and based on standard errors that are heteroskedasticity-robust. ***, **, and * indicate significance at the 1%, 5%, and 10% levels.

	1	2	3	4
Dependent variable:	SPILLOVER_PERSISTENCE			
Underlying measure:	ΔCoSP		ΔCoVaR	MES
Sample:	U.S. insurers	U.S. & CA insurers	U.S. insurers	
Window length:	Long		Baseline	
POST-KATRINA × EXPOSED	0.41***	0.57***	0.49**	0.89***
	[3.17]	[4.44]	[2.50]	[4.61]
Firm FE	Y	Y	Y	Y
Time FE	Y	Y	Y	Y
No. of firms	22	27	22	22
No. of obs.	506	621	495	475
Adj. R^2	0.895	0.895	0.792	0.855
Adj. R^2 within	0.009	0.014	0.006	0.038

Table IA.6. Spillover Persistence based on ΔCoVaR and Stock Market Bubbles.
This table presents OLS estimates analogously to those in Table 3 with the difference that SPILLOVER_PERSISTENCE is based on ΔCoVaR. t-statistics are shown in brackets and based on standard errors clustered at the firm and country-by-year levels. ***, **, and * indicate significance at the 1%, 5%, and 10% levels.

	1	2	3	4	5	6
Dependent variable:	SPILLOVER_PERSISTENCE (ΔCoVaR)		SPILLOVER PERSISTENCE$_{t+4}$	SPILLOVER_PERSISTENCE		
Sample:	Baseline			Within bubbles	Baseline	
BOOM	-5.43***	-1.56**	-1.09	4.39***	1.31	-0.43
	[-4.06]	[-2.04]	[-1.23]	[3.25]	[0.94]	[-0.31]
BUST	-2.38	1.09	-0.35		-0.51	0.84
	[-1.34]	[0.93]	[-0.27]		[-0.36]	[0.64]
BOOM × BURST_DISTANCE				-3.31***	-2.59***	-0.53
				[-7.65]	[-6.21]	[-1.16]
ΔCoVaR		0.67***				0.87***
		[2.75]				[3.18]
Macro controls	Y	Y	Y	Y	Y	Y
Market controls		Y			Y	Y
Firm characteristics		Y			Y	Y
Boom & bust length	Y	Y		Y	Y	Y
Boom & bust years						Y
Firm FE	Y	Y	Y	Y	Y	Y
Time FE		Y				Y
No. of firms	631	631	448	198	545	545
No. of obs.	5,983	5,983	4,380	840	4,961	4,961
Adj. R^2	0.174	0.286	0.149	0.385	0.233	0.296
Adj. R^2 within	0.057	0.044	0.049	0.291	0.121	0.050
p-value for H0: Same coeff on boom and bust	0.07	0.03	0.60			

Table IA.7. Robustness: Spillover Persistence and Crises.
Each column reports OLS regressions of banking crises indicators on systemic risk measures at the firm-year level:

$$y_{c,t} = \alpha X_{i,t} + \Gamma' C_{i,t} + \varepsilon_{i,t},$$

where $X_{i,t}$ is either SPILLOVER_PERSISTENCE or ΔCoVaR and $C_{i,t}$ is a vector of control variables and fixed effects for firm i in country c. Output loss is the % loss in GDP associated with banking crises, following Laeven and Valencia (2020). All crisis indicators are multiplied by 100 for readability. Variable definitions are analogous to those in Table 4. t-statistics are shown in brackets and based on standard errors clustered at the firm and country-by-year levels. ***, **, and * indicate significance at the 1%, 5%, and 10% levels.

	1	2	3	4	5	6	7
Dependent variable:	$100\times$ $1\{\text{CRISIS}_{t+1}\}$		$100\times$ $1\{\text{CRISIS}_{t+3}\}$	$100\times$ $1\{\text{CRISIS}_{t+1}\}$		$100\times$ $1\{\text{SYSTEMIC CRISIS}_{t+1}\}$	OUTPUT LOSS_{t+1}
ΔCoVaR	17.51***	-3.26***					
	[5.63]	[-3.10]					
SPILLOVER_PERSISTENCE		-0.49***	-0.22**			-0.36***	-0.16***
		[-3.61]	[-2.14]			[-3.19]	[-3.78]
SPILLOVER_PERSISTENCE (ΔCoVaR)				0.45*			
				[1.77]			
SPILLOVER_PERSISTENCE (MES)					0.22		
					[1.11]		
Macro controls		Y	Y	Y	Y	Y	Y
Market controls		Y	Y	Y	Y	Y	Y
Firm characteristics		Y	Y	Y	Y	Y	Y
Bank characteristics		Y	Y	Y	Y	Y	Y
AVERAGE_ΔCoSP/ΔCoVaR/MES		Y	Y	Y	Y	Y	Y
Firm FE	Y	Y	Y	Y	Y	Y	Y
Time FE		Y	Y	Y	Y	Y	Y
No. of firms	633	633	600	598	590	620	633
No. of obs.	6,833	6,833	6,031	5,943	5,614	6,722	6,833
Adj. R^2	0.159	0.745	0.755	0.063	0.087	0.623	0.702
Adj. R^2 within	0.132	0.314	0.246	0.022	0.038	0.318	0.371

Table IA.8. Robustness with Prewhitened CoSP: Spillover Persistence and Financial Conditions.

This table presents OLS estimates using prewhitened CoSP analogously to those in Table 2. t-statistics are shown in brackets and based on standard errors clustered at the firm level in columns (1) and (2) and at the firm and country-by-year levels in column (3). Standard errors in columns (4) and (5) are heteroscedasticity-consistent. ***, **, and * indicate significance at the 1%, 5%, and 10% levels.

	1	2	3	4	5
	(A) Macro-financial conditions			(B) Fire sales	
Dependent variable:	SPILLOVER_PERSISTENCE (prewhitened)				
Sample:	US		Full	US insurers	
NFCI	4.58***	4.26***			
	[19.57]	[14.69]			
CRISIS		1.45***	4.14***		
		[5.67]	[5.73]		
CREDIT_GROWTH		-0.67***	-0.05		
		[-10.67]	[-0.95]		
3M_YIELD_CHANGE		1.04***	0.51*		
		[10.72]	[1.84]		
TERM_SPREAD_CHANGE		0.56***	0.28		
		[6.77]	[1.19]		
CREDIT_SPREAD_CHANGE		0.67***	0.26		
		[10.68]	[1.44]		
POST-KATRINA $\times$ EXPOSED				0.16***	0.16**
				[2.79]	[2.43]
POST-KATRINA				-0.16***	
				[-2.79]	
Firm FE	Y	Y	Y	Y	Y
Time FE					Y
Standardized coefficients					
NFCI	.298	.278			
No. of firms	206	206	933	22	22
No. of obs.	2,753	2,753	9,986	286	286
Adj. R^2	0.141	0.228	0.162	0.980	0.981
Adj. R^2 within	0.095	0.186	0.051	0.045	0.007

Table IA.9. Robustness with Prewhitened CoSP: Spillover Persistence and Stock Market Bubbles.

This table presents OLS estimates using prewhitened CoSP analogously to those in Table 3. t-statistics are shown in brackets and based on standard errors clustered at the firm and country-by-year levels. ***, **, and * indicate significance at the 1%, 5%, and 10% levels.

	1	2	3	4	5	6
Dependent variable:	SPILLOVER_PERSISTENCE (prewtd)		SPILLOVER PERSISTENCE$_{t+4}$ (prewtd)	SPILLOVER_PERSISTENCE (prewtd)		
Sample:	Baseline			Within bubbles	Baseline	
BOOM	-4.84***	-1.90***	-3.44**	2.61	3.62**	0.23
	[-3.03]	[-2.78]	[-2.59]	[1.62]	[2.02]	[0.23]
BUST	-2.22	-0.21	-1.34		-0.26	1.10
	[-1.26]	[-0.25]	[-1.43]		[-0.20]	[1.33]
BOOM × BURST_DISTANCE				-1.95***	-2.96***	-0.78**
				[-4.63]	[-5.37]	[-1.99]
ΔCoVaR		0.08				0.20
		[0.45]				[0.98]
Macro controls	Y	Y	Y	Y	Y	Y
Market controls		Y			Y	Y
Firm characteristics		Y			Y	Y
Boom & bust length	Y	Y		Y	Y	Y
Boom & bust years						Y
Firm FE	Y	Y	Y	Y	Y	Y
Time FE		Y				Y
No. of firms	665	665	456	232	575	575
No. of obs.	6,975	6,975	4,835	1,026	5,773	5,773
Adj. R^2	0.235	0.464	0.105	0.454	0.329	0.494
Adj. R^2 within	0.114	0.050	0.039	0.332	0.211	0.074
p-value for H0: Same coeff on boom and bust	0.10	0.09	0.12			

Table IA.10. Robustness with Prewhitened CoSP: Spillover Persistence and Fragility in the Financial System.

This table presents OLS estimates using prewhitened CoSP analogously to those in Table 4. t-statistics are shown in brackets and based on standard errors clustered at the firm and country-by-year levels. Standardized coefficients are the change in the dependent variable for a standard deviation change in SPILLOVER_PERSISTENCE. ***, **, and * indicate significance at the 1%, 5%, and 10% levels.

	1	2	3	4	5	6	7	8
Dependent variable:		LEVERAGE_{t+1}		CDS_{t+1}			$100 \times 1\{\text{CRISIS}_{t+1}\}$	
Sample:	Baseline		Ban & Bro				Baseline	
SPILLOVER_PERSISTENCE (prewtd)	-0.06**	-0.08	-0.02	-0.00	-0.01**	-0.67**	-0.38***	-0.39***
	[-2.14]	[-1.64]	[-0.41]	[-1.09]	[-2.41]	[-2.07]	[-3.31]	[-3.40]
SPILLOVER_PERSIST. × SIZE			0.07**		0.01**			
			[2.15]		[2.21]			
SPILLOVER_PERSIST. × LEVERAGE			-0.15**		-0.01			
			[-2.45]		[-1.65]			
SPILLOVER_PERSIST. × MARKET-TO-BOOK			-0.08**		-0.02***			
			[-2.49]		[-3.07]			
SPILLOVER_PERSIST. × LIQUIDITY_RATIO			0.05		0.02*			
			[1.48]		[1.87]			
SPILLOVER_PERSIST. × DEMAND_DEPOSITS			-0.04		-0.01			
			[-1.50]		[-1.38]			
SPILLOVER_PERSIST. × TIME_DEPOSITS			0.03		-0.00			
			[1.00]		[-0.34]			
SPILLOVER_PERSIST. × IMPAIRED_LOANS			-0.09***		-0.03***			
			[-2.77]		[-2.98]			
SPILLOVER_PERSIST. × INTANGIBLE_ASSETS			-0.08***		-0.01*			
			[-2.81]		[-1.92]			
ΔCoVaR								-3.13***
								[-3.02]
Macro controls	Y	Y	Y	Y	Y		Y	Y
Market controls		Y	Y		Y		Y	Y
Firm characteristics	Y	Y	Y	Y	Y		Y	Y
Bank characteristics		Y	Y	Y	Y		Y	Y
AVERAGE_ΔCoSP	Y	Y	Y	Y	Y	Y	Y	Y
Firm FE	Y	Y	Y	Y	Y	Y	Y	Y
Time FE		Y	Y	Y	Y		Y	Y
No. of firms	790	189	189	76	76	625	625	625
No. of obs.	8,276	1,603	1,603	671	671	6,667	6,667	6,667
Adj. R^2	0.722	0.857	0.861	0.815	0.832	0.163	0.744	0.745
Adj. R^2 within	0.186	0.167	0.189	0.097	0.181	0.136	0.308	0.312
Standardized coefficient:	-0.03	-0.04	-0.01	-0.03	-0.10	-0.11	-0.06	-0.06

D.2 Liquidity and Autocorrelation of Stock Returns

Daily turnover by value (VA) and volume (VO) are from Thomson Reuters Datastream at the security-day-level. $VO_{i,t}$ is the median daily turnover by volume (in thd USD) for firm i's common equity in time period t. The Amihud measure is defined by (see Amihud, 2002)

$$\text{ILLIQ}_{i,t} = \frac{1}{n_t} \sum_{\tau=1}^{n_t} \frac{|r_{i,t,\tau}|}{VA_{i,t,\tau}}, \tag{IA.7}$$

where n_t is the number of observations in time period t, $r_{i,t,\tau}$ is the daily return and $VA_{i,t,\tau}$ the turnover by value in thd USD on day τ in time period t for firm i's common equity. To calculate the turnover by volume of the system, I use the average daily turnover volume across firms in the system. The Amihud measure for the system is based on the system's value-weighted return and average daily turnover by value. Finally, I winsorize all variables at the 1% and 99% levels.

Table IA.11. Spillover Persistence and Stock Market Liquidity.
This table reports estimates from OLS panel regressions of SPILLOVER_PERSISTENCE based on ΔCoSP in columns (1) to (4) and of AVERAGE_ΔCoSP in columns (5) to (8) at the firm-year level. The explanatory variables are a financial institution's and the system's stock market turnover in columns (1), (2), (5), and (6), and the financial institution's and system's Amihud measure for illiquidity in columns (3), (4), (7), and (8). t-statistics are shown in brackets and based on standard errors clustered at the firm and country-by-year levels. ***, **, and * indicate significance at the 1%, 5%, and 10% levels.

	1	2	3	4	5	6	7	8
Dependent variable:	SPILLOVER_PERSISTENCE				AVERAGE_ΔCoSP			
log(FIRM_TURNOVER)	0.20	-0.16			0.01***	0.00***		
	[0.78]	[-0.85]			[4.30]	[3.88]		
log(SYSTEM_TURNOVER)	2.46***	1.27***			0.01***	0.00**		
	[8.57]	[2.96]			[8.73]	[2.12]		
FIRM_ILLIQ			-0.00	-0.00			-0.00***	-0.00**
			[-1.50]	[-0.61]			[-2.74]	[-2.04]
SYSTEM_ILLIQ			-3.66	-3.07			-0.04**	-0.02*
			[-0.88]	[-0.96]			[-2.25]	[-1.79]
Firm FE	Y	Y	Y	Y	Y	Y	Y	Y
Time FE		Y		Y		Y		Y
No. of firms	935	935	728	728	935	935	728	728
No. of obs.	10,179	10,179	6,008	6,008	10,179	10,179	6,008	6,008
Adj. R^2	0.214	0.410	0.147	0.385	0.321	0.714	0.181	0.683
Adj. R^2 within	0.097	0.006	0.003	0.001	0.202	0.018	0.013	0.005

To examine the relation between SPILLOVER_PERSISTENCE and the auto-serial correlation of stock prices, I estimate the autocorrelation function of the system's return for each estimation window. Then, I regress CoSP measures on the average autocorrelation coefficient across lags of 1 to 10 days. Table IA.12 reports the estimates. There is neither a significantly positive correlation between the level of autocorrelation and SPILLOVER_PERSISTENCE nor AVERAGE_ΔCoSP.

Table IA.12. Spillover Persistence and Stock Return Autocorrelation.

This table reports estimates from OLS panel regressions of SPILLOVER_PERSISTENCE based on ΔCoSP in columns (1) and (2) and of AVERAGE_ΔCoSP in columns (3) and (4) at the firm-year level. The explanatory variable is the average (across 1 to 10-day lags) autocorrelation of the system's stock returns. t-statistics are shown in brackets and based on standard errors clustered at the firm and country-by-year levels. ***, **, and * indicate significance at the 1%, 5%, and 10% levels.

	1	2	3	4
Dependent variable:	SPILLOVER_PERSISTENCE		AVERAGE_ΔCoSP	
$ACF_{1:10}$	-145.83***	-43.22	-1.12***	-0.18
	[-4.87]	[-0.94]	[-7.36]	[-1.20]
Firm FE	Y	Y	Y	Y
Time FE		Y		Y
No. of firms	938	938	938	938
No. of obs.	10,234	10,234	10,234	10,234
Adj. R^2	0.180	0.402	0.307	0.709
Adj. R^2 within	0.057	0.001	0.187	0.003

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