Internet Appendix for
Life Insurance Convexity[*]

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[*]The views expressed in this paper are the authors' and do not necessarily reflect those of the European Central Bank or the Eurosystem.

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A Surrender Options in the U.S.

In the U.S., surrender payouts (including full and partial withdrawals) are similarly large as in Europe, amounting to EUR 308 billion (equivalently, $345 billion) in 2019, which corresponds to roughly 44% of total life insurance payouts (NAIC, 2020). U.S. life insurance products with cash value also entail surrender options. These products include universal life and whole life insurance as well as variable and deferred annuities (Berends et al., 2013).

For individual deferred annuities, the surrender value is mandated to correspond to at least 87.5% of the accumulated gross cash value up to the surrender date and additional interest credits less surrender charges (NAIC, 2017). Similar to German life insurance policies, the guaranteed minimum interest rate is determined at contract origination.[1] Therefore, there exists a minimum guaranteed surrender value that is independent of market developments.

For multi-year deferred annuities, the surrender value is typically subject to a market value adjustment (MVA), at least in the first contract years. This can cause both upward and downward changes based on market developments (NAIC, 2021). The MVA compares interest rates at contract origination with rates at the surrender date. If interest rates have increased (decreased) during the active contract period, the effect of the MVA on the surrender value will be negative (positive), i.e., the policyholder will receive relatively less (more).

Variable annuities come with a broad flexibility for policyholders to decide on the underlying investment (typically mutual funds) and on guarantee components (Koijen and Yogo, 2022). Depending on the chosen financial guarantee, surrender values may react less sensitive to an interest rate rise than the underlying investment, which strengthens surrender incentives similarly as for the contracts we study in our model.

Surrender penalties for U.S. life insurance contracts are typically up to 10% of the con-

[1] The guaranteed minimum interest rate must be between 1 and 3% and, within this range, depends on the five-year U.S. Constant Maturity Treasury yield reduced by 125 bps (NAIC, 2017).

tract's cash value in the first year and then decrease by 100 bps annually. Often, 10% of the cash value can be withdrawn without a penalty in the first contract years.

B Anecdotal Evidence

Anecdotal evidence emphasizes the interaction of market interest rates, surrender options, and life insurers' liquidity risk. We highlight three historical examples. First, in response to rising U.S. market interest rates in the late 1970s and early 1980s, U.S. surrender rates increased sharply from roughly 3% in 1951 to 12% in 1985 (Kuo et al., 2003). As a result, U.S. life insurers liquidated a large share of their investments (Russell et al., 2013).

Second, the surrender of guaranteed investment contracts (GICs), which are savings contracts with financial guarantees resembling modern savings contracts, significantly contributed to U.S. life insurer failures in the 1990s (Brewer et al., 1993; Jackson and Symons, 1999; Brennan et al., 2013). Rising interest rates in particular sparked mass surrenders of GICs sold by *General American*, a U.S. life insurer, resulting in its failure in 1999 (Fabozzi, 2000; Brennan et al., 2013).

Third, rising interest rates also triggered large surrenders in South Korea in 1997–1998. As interest rates sharply rose (by approximately 4 ppt for 5-year government bonds within a few months), annualized surrender rates increased from 11% to 54.2% for long-term savings contracts, and life insurers' gross premium income fell by 26%. Life insurers were forced to liquidate assets, and approximately one-third of them exited the market (Geneva Association, 2012).

C Empirical Analysis: Data and Additional Results

C.1 Data

Table IA.1. Variable definitions and data sources.

Note: *BaFin* refers to data retrieved from the "Erstversicherungsstatistik" of the German financial supervisory authority *BaFin*, available either in print or online at `https://www.bafin.de/DE/PublikationenDaten/Statistiken/Erstversicherung/erstversicherung_artikel.html`. *GDV* refers to data shared with us by the German association of insurers.

Variable	**Definition**
Insurer-Year level	
Surrender rate	Fraction of life insurance contracts surrendered weighted by contract volume (*Source: BaFin*)
New business	Volume of new insurance business relative to that of total insurance business at the previous year's end (*Source: BaFin*)
Year level	
Interest rate	10-year German government bond rate (*Source: Bundesbank*)
Guaranteed return	Annually guaranteed minimum return for new German life insurance contracts (*Source: http://gdv.de*)
Contract return	Average market-wide realized contract return for traditional endowment contracts in Germany (*Source: Assekurata*)
log(New German contracts)	Logarithm of the number of new German life insurance contracts (*Source: GDV*)
New term life	Fraction of new term life insurance contracts relative to all new life insurance contracts in Germany (*Source: GDV*)
Inflation	Annual change in German CPI (*Source: BIS*)
GDP growth	Annual change in German GDP (*Source: OECD*)
Investment growth	Annual change in German investment (*Source: OECD*)
Crisis	Indicator for banking crises (*Source: Laeven and Valencia (2018)*)
MoPoSurp	End-of-year cumulative U.S. monetary policy shocks, computed as the sum of past monetary policy surprises (since 1990), which are defined following Jarocínski and Karadi (2020) as the first principal component of the surprises in interest rate derivatives with maturities from 1 month to 1 year, which are measured as described in Gürkaynak et al. (2005) (*Source: http://marekjarocinski.github.io*)
Pure MoPoSurp	End-of-year cumulative U.S. monetary policy shocks (since 1990) purged of central bank information shocks with simple ("Poor Man's") sign restrictions as described by Jarocínski and Karadi (2020) (*Source: http://marekjarocinski.github.io*)

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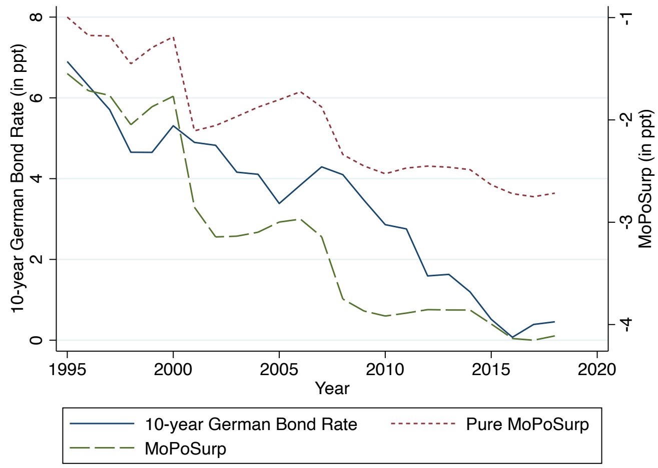
Table IA.1 – *Continued from previous page*

Variable	**Definition**
CB InfoSurp	End-of-year cumulative U.S. central bank information shocks (since 1990) obtained using simple ("Poor Man's") sign restrictions as described by Jarocínski and Karadi (2020) (*Source:* `http://marekjarocinski.github.io`)
%U.S. Imports	U.S. Imports of Goods by Customs Basis from Germany / (U.S. Imports of Goods by Customs Basis from Germany + U.S. Exports of Goods by F.A.S. Basis to Germany) (*Source: FRED St. Louis*)

Figure IA.1. German government bond rates and U.S. monetary policy surprises.
The figure plots the evolution of the 10-year German government bond rate (left axis), cumulative monetary policy surprises (right axis), and pure cumulative monetary policy surprises (right axis), which are purged from central bank information surprises following Jarocínski and Karadi (2020), from 1995 to 2018.



When processing data from BaFin's Erstversicherungsstatistik, we use the following conventions:

1. We translate values from the historical German currency ("Deutsche mark") to the euro for the years 1995 to 2000 using the official exchange rate 1 EUR = 1.95583 Deutsche marks.
2. We infer the overall surrender rate for years $t \leq 2015$ (for which it is not directly reported) as

$$\bar{\lambda}_{i,t} = \frac{\text{insurance in force}_{i,t-1} \cdot \lambda_{i,t}^{\text{late}} + \text{new business}_{i,t-1} \cdot \lambda_{i,t}^{\text{early}}}{(\text{insurance in force}_{i,t-1} + \text{insurance in force}_{i,t})/2},$$

   where insurance in force$_{i,t-1}$ is insurance in force at year-end $t-1$ or, equivalently, insurance in force at year-begin t of insurer i, and $\lambda_{i,t}^{\text{early}}$ and $\lambda_{i,t}^{\text{late}}$ are the surrender rates for new and old business, respectively.
3. The late surrender rate is defined as the share of the total sum insured of contracts that are (partially or fully) surrendered and involve a positive surrender payout (including

lapses, on which policyholders stop paying premiums but retain a positive sum insured) relative to the total sum insured at year begin. The variable is available until 2015.

4. The early surrender rate is defined as the share of the total sum insured of contracts that are prematurely terminated and do not involve a positive surrender payment or a positive sum insured remaining (which predominantly applies to new contracts) relative to the total sum insured of newly sold contracts. The variable is available until 2015.

To construct the annual German government bond rate, we retrieve end-of-month yields from the German Bundesbank and take annual averages.

C.2 Additional Results

Table IA.2. Surrender Rates and Interest Rates: Robustness with Average Future Contract Return.

This table estimates the specifications from Table 2, controlling for the average contract return in future years $t+1$ and $t+2$, denoted Contract return$_{t+(1:2)}$. t-statistics are shown in brackets, based on standard errors that are clustered at the insurer level. ***, **, * indicate significance at the 1%, 5% and 10% level.

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:	Surrender rate$_{i,t}$					
	OLS			IV		
Sample:	Full	Young contracts	Full		Young contracts	Full
Interest rate$_{t-1}$	0.25***	0.60**		0.10	0.88**	
	[4.36]	[2.24]		[0.92]	[2.16]	
Contract return$_{t+(1:2)}$	-0.01	-0.27		0.12	-0.13	
	[-0.07]	[-1.63]		[1.07]	[-0.54]	
Interest rate$_{t-1}$ × Guarantee$_{t-1}$		-0.16*			-0.28**	
		[-1.97]			[-1.99]	
Guarantee$_{t-1}$		1.16***			1.41***	
		[3.89]			[3.90]	
Interest rate$_{t-1}$ × Guarantee$_{t-1}$ × New business$_{i,t-1}$			-0.02***			-0.02***
			[-3.93]			[-3.34]
Macro controls	Y	Y		Y	Y	
New business$_{i,t-1}$	Y	Y	Y	Y	Y	Y
Interest rate$_{t-1}$ × New business$_{i,t-1}$			Y			Y
Guarantee$_{t-1}$ × New business$_{i,t-1}$			Y			Y
Insurer FE	Y	Y	Y	Y	Y	Y
Year FE			Y			Y
First stage						
MoPoSurp$_{t-1}$				1.12***	2.85***	
				[45.33]	[37.30]	
MoPoSurp$_{t-1}$ × Guarantee$_{t-1}$ × New business$_{i,t-1}$						0.47**
						[2.45]
F Statistic				707	152	237
No. of obs.	2,251	1,121	2,251	2,251	1,121	2,251
No. of insurers	160	137	160	160	137	160

Table IA.3. Surrender Rates and Interest Rates: Robustness.

This table presents estimates from regressions of insurer-level annual surrender rates on the 10-year German government bond rate from 1996 to 2019. Columns (1) to (4) are based on the model

$$\text{Surrender rate}_{i,t} = \alpha\text{Interest rate}_{t-1} + \beta\text{New business}_{i,t-1} + \xi Y_{t-1} + u_i + \varepsilon_{i,t}.$$

Column (1) uses pure monetary policy surprises as an instrument for 10-year German government bond rates and additionally controls for central bank information shocks. Column (2) uses the 10-year U.S. treasury rate as an instrument for 10-year German government bond rates. Columns (3) and (4) present reduced-form estimates. Columns (1) to (3) control for the lagged share of U.S. imports from Germany relative to the sum of U.S. imports and U.S. exports from/to Germany in addition to the controls in Table 2. Column (4) additionally controls for the 10-year German government bond rate. Columns (5) and (6) regress annual changes in surrender rates on annual changes in the 10-year German government bond rate, both from $t-1$ to t, in the following specification:

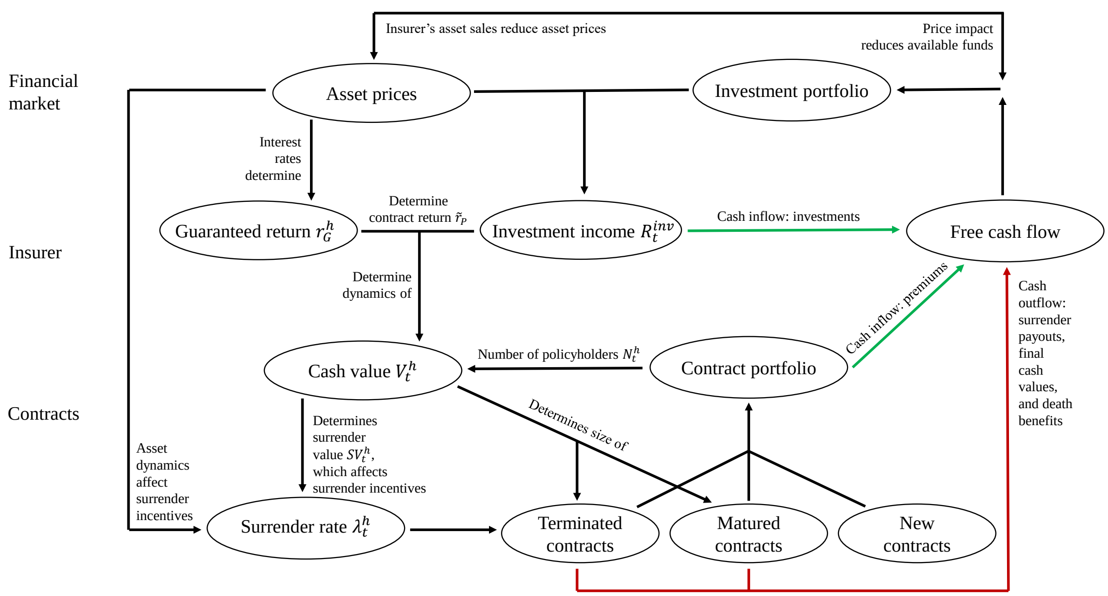
$$\Delta\text{Surrender rate}_{i,t} = \alpha\Delta\text{Interest rate}_{t} + \beta\text{New business}_{i,t-1} + \xi Y_{t-1} + u_i + \varepsilon_{i,t}.$$

$1\{\Delta\text{Interest rate}_t > 0\}$ is an indicator for an increase in the 10-year German government bond rate from $t-1$ to t. The sample is at the insurer-by-year level from 1996 to 2019. Y_{t-1} is a vector with the same macroeconomic control variables as in Table 2. Detailed variable definitions and data sources are reported in the Internet Appendix. t-statistics are shown in brackets, based on standard errors that are clustered at the insurer level. ***, **, * indicate significance at the 1%, 5% and 10% level.

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:	Surrender rate				ΔSurrender rate	
	IV		OLS			
Interest rate$_{t-1}$	0.224***	0.291***		0.315***		
	[3.21]	[5.54]		[5.73]		
CB InfoSurp$_{t-1}$	0.115					
	[0.40]					
%U.S. Imports$_{t-1}$	1.289	2.500	-1.700			
	[0.86]	[1.35]	[-1.37]			
MoPoSurp$_{t-1}$			0.337***	-0.162		
			[4.01]	[-1.24]		
ΔInterest rate$_t$					0.166***	0.144**
					[4.41]	[2.07]
$1\{\Delta\text{Interest rate}_t > 0\} \times \Delta\text{Interest rate}_t$						0.515***
						[2.61]
$1\{\Delta\text{Interest rate}_t > 0\}$						-0.145
						[-1.55]
Macro controls	Y	Y	Y	Y	Y	Y
New business$_{i,t-1}$	Y	Y	Y	Y	Y	Y
Insurer FE	Y	Y	Y	Y	Y	Y
First stage						
Pure MoPoSurp$_{t-1}$	2.25***					
	[182.11]					
U.S. treasury rate$_{t-1}$		0.97***				
		[248.09]				
F Statistic	4,376	5,696				
No. of obs.	2,251	2,251	2,251	2,251	2,064	2,064
No. of insurers	160	160	160	160	151	151

D Model and Calibration Details

Figure IA.2. Illustration of Key Model Ingredients and Dynamics.
The financial market model determines asset prices and, in particular, government bond rates, which determine the guaranteed return for the new cohort of contracts in year h, r_G^h. Jointly with the insurer's investment portfolio, asset prices also determine the insurer's investment income R_t^{inv}. A fraction ν of the investment income is passed on to policyholders. The maximum of the guaranteed return and the policyholder's fraction of the investment income determines the contract return $\tilde{r}_P$, which drives the dynamics of life insurance contracts' cash value V_t^h. The cash value determines the surrender value SV_t^h. Surrender decisions are based on comparing SV_t^h with the present value of the contract m_t^h, resulting in the surrender rate λ_t^h. Cash values also determine the size of surrendered and matured contracts. Contract portfolio dynamics are jointly determined by the volume of terminated, matured, and new contracts, reflected in the number of policyholders N_t^h of cohort h. Contracts may be terminated either due to surrenders, upon which the surrender value is paid, or policyholder death, upon which a fixed death benefit is paid. The insurer's total free cash flow is given by the sum of investment income and premiums net of cash outflows due to terminated and matured contracts. Excess cash is reinvested, whereas a negative free cash flow forces the insurer to sell assets. Asset sales reduce asset prices and, thereby, negatively impact the funds available for reinvestment.

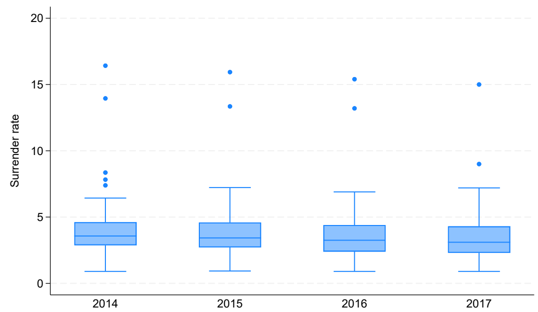


D.1 Calibration of Surrender Decisions

We calibrate the model of contract surrenders by exploiting the cross-sectional distribution of German life insurance surrender rates in the Erstversicherungsstatistik (described in Section 3). The first period of simulated surrenders in our model (between year-ends $t = 0$ and

$t = 1$) corresponds to the year 2016. Because the Erstversicherungsstatistik separately includes early and late surrender rates only until 2015, we use data from 2015. In Figure IA.3, we show that the distribution of the insurer-level surrender rate (averaged across all cohorts) is similar in 2015 and 2016, which is consistent with the then very stable German economic environment and interest rates in particular.

Figure IA.3. Distribution of Surrender Rates across German Life Insurers.



We calibrate the model's parameters $\beta = (\beta_0, \beta_1, \beta_2)$ by solving the following optimization problem:

$$\min_{\beta} \left(\sum_{i:\text{low } r_A} \sum_{h} \frac{\hat{w}_{i,h}}{\sum_{j:\text{low } r_A} \sum_g \hat{w}_{j,g}} \hat{\lambda}^{early}_{2015,i} - \lambda^0_1(\beta, \delta_{\text{low}}) \right)^2 + \left(\sum_{i:\text{high } r_A} \sum_{h} \frac{\hat{w}_{i,h}}{\sum_{j:\text{high } r_A} \sum_g \hat{w}_{j,g}} \hat{\lambda}^{early}_{2015,i} - \lambda^0_1(\beta, \delta_{\text{high}}) \right)^2 \quad \text{(IA.1)}$$

$$s.t. \sum_{i,h} \frac{\hat{w}_{i,h}}{\sum_{j,g} \hat{w}_{j,g}} \hat{\lambda}^h_{2015,i} = \sum_h w_h \lambda^h_1(\beta) \quad \text{(IA.2)}$$

$$\sum_{i,h} \frac{\hat{w}_{i,h}}{\sum_{j,g} \hat{w}_{j,g}} \hat{\lambda}^{early}_{2015,i} = \lambda^0_1(\beta). \quad \text{(IA.3)}$$

$\sum_{i,h} \frac{\hat{w}_{i,h}}{\sum_{j,g} \hat{w}_{j,g}} \hat{\lambda}^h_{2015,i}$ is the average realized surrender rate across all German life insurers i and cohorts h in 2015 (3.58%). Cohorts as well as insurers are weighted by the total volume of insurance in force at year-begin, $\hat{w}_{i,h}$, of cohort h of insurer i.[2] (IA.2) requires that the

[2] Insurance in force (*Versicherungssumme*) is the guaranteed amount to be paid out if the policyholder does not surrender. We compute insurance in force in our model as the sum of guaranteed savings (including

average realized surrender rate coincides with the average surrender rate in the first year of the model, $\lambda_1^h(\beta)$, weighted across cohorts by insurance in force, w_h.

$\hat{\lambda}_{2015,i}^{early}$ is the realized early surrender rate (for young cohorts) of insurer i.[3] (IA.3) requires that the average realized early surrender rate (8.56%) coincides with the average surrender rate of the youngest cohort ($h = 0$) in the first year of the model, both weighted by insurance in force.

The objective function (IA.1) minimizes the deviation between model-implied and realized surrender rates when varying the (expected) contract return. Because contract returns are not reported in the Erstversicherungsstatistik and expected future contract returns are not observable, we use information on realized investment returns, $\hat{r}_{A,2015,i}$, instead. We denote by $\Delta = \hat{r}_{P,2015} - \sum_{i,h} \frac{\hat{w}_{i,h}}{\sum_{j,g} \hat{w}_{j,g}} \hat{r}_{A,2015,i}$ the difference between the average contract return in the German life insurance market in 2015 (3.16%) and the volume-weighted average investment return in 2015 (3.51%). We denote by $\{i : \text{low } r_A\}$ the set of life insurers in the 2nd volume-weighted quartile of investment returns and by $\hat{r}_{A,\text{low}} = \sum_{i:\text{low } r_A} \sum_h \frac{\hat{w}_{i,h}}{\sum_{j:\text{low } r_A} \sum_g \hat{w}_{j,g}} \hat{r}_{A,2015,i}$ the volume-weighted average investment return of these insurers (3.31%). The corresponding volume-weighted average early surrender rate is 9.37%. Analogously, we define by $\{i : \text{high } r_A\}$ insurers in the 3rd quartile of investment returns and by $\hat{r}_{A,\text{high}}$ their average investment return (3.83%), with the corresponding volume-weighted average early surrender rate of 6.64%. Then, $\delta_{\text{low}} = \Delta + \hat{r}_{A,\text{low}}$ and $\delta_{\text{high}} = \Delta + \hat{r}_{A,\text{high}}$ approximate the average contract returns of these insurers.[4] Finally, we compute "shocked" surrender rates by shifting the annual future contract returns in Equation (4) by $\delta_{\text{low}} - \tilde{r}_{P,0}$

future premiums) and the current one-year mortality component, such that insurance in force in cohort h is equal to

$$V_t^h (1 + r_G^h)^{T^h - t} + N_t^h \sum_{\tau=1}^{T^h - t - 1} (P^h - q_{t+\tau}^h v_m)(1 + r_G^h)^{T^h - t - \tau} + N_t^h v_m. \tag{IA.4}$$

[3] We truncate $\hat{\lambda}_{2015,i}^{early}$ at 0.3 to remove the impact of outliers.

[4] These estimates are particularly accurate when guaranteed returns are not binding, which is the case in the low-interest rate environment of 2015.

and $\delta_{\text{high}} - \tilde{r}_{P,0}$, while holding all else constant, where $\tilde{r}_{P,0}$ is the contract return in the model in $t = 0$.

The resulting calibration is $\beta = (\beta_0, \beta_1, \beta_2) = (0.0574, 1.0148, 0.365)$.

D.2 Accounting of Insurance Liabilities

Under European statutory accounting following the Solvency II regulation, insurance liabilities reflect the market-consistent value of contracts. For this purpose, insurers compute a *best estimate* of market-consistent contract values. We compute the Solvency II balance sheet mainly to scale our model to the size of European life insurers. We approximate the value of liabilities in cohort h at time t on the Solvency II balance sheet as follows (note that future mortality payouts are covered by future premiums by assumption and, thus, do not enter the present value of liabilities):

$$\begin{aligned} PV_t^h =& V_t^h \Bigg(\sum_{j=1}^{T^h - t} \frac{\vartheta \lambda_t^h (1-\lambda_t^h)^{j-1} \prod_{\tau=1}^{j-1} (1 - q_{t+\tau-1}^h)(1 + \max\{r_G^h, \hat{r}_{P,t+\tau}^*\})}{(1 + r_{f,t,j-1})^{j-1}} \\ &+ \frac{(1-\lambda_t^h)^{T^h - t} \prod_{\tau=1}^{T^h - t} (1 - q_{t+\tau-1}^h)(1 + \max\{r_G^h, \hat{r}_{P,t+\tau}^*\})}{(1 + r_{f,t,T^h - t})^{T^h - t}} \Bigg) + \frac{q_t^h N_t^h v_m}{1 + r_{f,t,1}}. \qquad \text{(IA.5)} \end{aligned}$$

Here, we make two assumptions. The first is that the most recent realized surrender rate λ_t^h in cohort h is used for future years. The second is that the future profit participation rate, $\hat{r}_{P,t+\tau}^h$, is estimated from a log-linear model. In particular, at each year, the profit participation rate $\tilde{r}_t^*$ is fitted to a log-linear model, which is then used to predict future profit participation rate: $\tilde{r}_i^* = \alpha + \beta \log(10 + i - t) + \varepsilon_i$, which is estimated using OLS based on observations from the past 10 years, $i = t - 9, ..., t$. Then, the predicted profit participation rate is given by $\hat{r}_i^* = \hat{\alpha} + \hat{\beta} \log(10 + i - t)$ for $i > t$.

PV_t^h affects the main results in two ways. First, we calibrate the leverage of the insurer's initial balance sheet based on the value of liabilities implied by PV_t^h. This is the reason for using the log-linear model above to approximate future profit participation rates rather than simulated future profit participation rates, which require the calibrated balance sheet as input. Second, the insurer defaults if the market value of total assets drops below $\sum_h PV_t^h$, in which case contract returns drop to zero.

D.3 Calibration of the Initial Contract Portfolio

To calibrate the initial cash value of contract cohorts, we use the following data:

- the volume of life insurance savings contracts ("Kapitalversicherungen") newly issued in year h, N^h, obtained from the German insurance association, GDV (in million EUR)[5],
- the life insurance sector's surrender rate, $\tilde{\lambda}_t$,
  - 1996–2015: for the median German life insurer (weighted across insurers by contract portfolio size) according to BaFin's Erstversicherungsstatistik
  - 1976–1995: the average surrender rate reported by the German insurance association, GDV, scaled by the ratio of the BaFin surrender rate to the GDV surrender rate from 1996 to account for differences in the underlying set of life insurers
- the realized contract return of German life insurance contracts
  - 1996–2015: reported by Assekurata, a rating agency for German life insurers[6]
  - 1976–1995: predicted by fitting a linear model to the average contract return reported by Assekurata for 1996–2015 using the 10-year moving average of 5-year German government bond rates reported in the IMF's International Financial Statistics as explanatory variable (the R^2 is 91%). We use bond rates from the IMF's statistics because of the long time series available.

[5] We thank the GDV for sharing the data with us.
[6] We thank Assekurata for sharing the data with us.

Since the surrender rate and contract return are not available at the cohort level, we make the following assumptions: (1) within each cohort h, each contract pays a premium of EUR 1 each year if not surrendered or matured, (2) each contract has a lifetime of 40 years at inception, and (3) each contract's surrender rate in year t can be approximated by the average surrender rate $\tilde{\lambda}_t$. However, accumulating contracts since 1976 according to these assumptions must not necessarily arrive at the representative contract portfolio in 2015. Instead, contract dynamics might have deviated due to the presence of single premiums, heterogeneity in the surrender rate and contract return, and time-varying insurance supply.

To evaluate the representativeness of the initial contract portfolio, we use two key portfolio characteristics: the average guaranteed return per contract and the portfolio's modified duration.[7] Assekurata (2016) reports an average guaranteed return of 2.97% for German life insurers in 2015. The German association of insurers (GDV) reports a modified duration of liabilities of 14.1 for the median insurer and 14.8 for the weighted average in 2013. Following the assumptions above, our initial portfolio would exhibit a much shorter duration. In this case, the portfolio weight of older contracts (with a short remaining time to maturity and, thus, short duration) is too large. To offset this bias, we modify the size of cohorts $h \in \{-39, ..., 0\}$ as follows:

$$\hat{N}^h = \left[N^h \left(1 + g \cdot (h + T^h)\right)\right].$$

The larger the adjustment factor g, the larger is the volume of younger relative to older contracts. This increases the modified duration. We find that $g = 5$ lifts the modified duration

[7] Consistent with EIOPA (2016), we calculate a cohort's modified duration as

$$\frac{1}{(1+r_{f,t,T^h-t})PV_t^h}\Bigg[V_t^h\Bigg(\sum_{j=1}^{T^h-t}(j-1)\frac{\vartheta\lambda_t^h(1-\lambda_t^h)^{j-1}\prod_{\tau=1}^{j-1}(1-q_{t+\tau-1}^h)(1+\max\{r_G^h,\hat{r}_{P,t+\tau}^*\})}{(1+r_{f,t,j-1})^{j-1}}$$
$$+(T^h-t)\frac{(1-\lambda_t^h)^{T^h-t}\prod_{\tau=1}^{T^h-t}(1-q_{t+\tau-1}^h)(1+\max\{r_G^h,\hat{r}_{P,t+\tau}^*\})}{(1+r_{f,t,T^h-t})^{T^h-t}}\Bigg)+\frac{q_t^h N_t^h v_m}{1+r_{f,t,1}}\Bigg]$$

to 13.94 years and the average guaranteed return to 3.12%, which are both reasonably close to the empirical moments. Finally, we scale $\hat{N}^h$ by dividing it by $\hat{N}^0/10,000$ such that the implied number of new contracts at $t = 0$ is equal to 10,000.

D.4 Calibration of the Insurer's Investment Portfolio

We calibrate the insurer's asset portfolio weights based on GDV (2016), according to which German life insurers held 6.7% in stocks (shares and participating interests) and 3.9% in real estate in 2015. For the corporate bond portfolio weight, we aggregate German life insurers' investments in 2015 in mortgages (5.8%), loans to credit institutions (9.8%), loans to companies (1%), contract and other loans (0.5%), corporate bonds (10.3%), and subordinated loans and profit participation rights, call money, time and fixed deposits and other bonds and debentures (6.7%), which results in 34.1% and coincides with the fraction of corporate bonds reported by the EIOPA (2014) for German insurers. We allocate the remaining fraction of fixed-income instruments to government bonds (55.3%).

The weights within subportfolios are based on Berdin et al. (2017) and EIOPA (2014) and reported in Table IA.4. We include a large home bias toward German government bonds, which, however, has little impact on our results. Due to the absence of more granular data, we calibrate real estate and stock weights to yield a plausible home bias of 60% for German real estate and stocks and equally distribute the remaining weights.

Bond maturities differ within the insurer's portfolio, such that within each bond category, the oldest bond is due in 1 year, the youngest government bond is due in 20 years, and the youngest corporate bond is due in 10 years, reflecting the longer duration of government bonds in insurers' portfolios. Bond coupons are based on the (government or corporate) bond yield at bond issuance.

To calibrate the modified duration of different asset classes, we use 9.3 years as a benchmark duration for the fixed-income portfolio, based on the stress test results in EIOPA (2016,

Table IA.4. Investment Portfolio Allocation.
The table depicts the weights and average modified duration of each asset class in the insurer's investment portfolio. The calibration is based on EIOPA (2014, 2016) and GDV (2016).

Entire Investment Portfolio	**Weight**	**Duration**
Government Bonds	55.3%	10.4
Corporate Bonds	34.1%	7.5
Stocks	6.7%	-
Real Estate	3.9%	-
Government Bond Portfolio	**Weight**	**Modified Duration**
German/All Government Bonds	90.4%	10.45
French/All Government Bonds	2.4%	10.12
Dutch/All Government Bonds	2.4%	10.45
Italian/All Government Bonds	2.4%	8.03
Spanish/All Government Bonds	2.4%	10.45
Corporate Bond Portfolio	**Weight**	**Duration**
AAA/All Corporates	23.6%	7.36
AA/All Corporates	16.85%	8.08
A/All Corporates	33.71%	7.65
BBB/All Corporates	25.84%	7.22

Table 6) (9.6 years for 2015) and EIOPA (2014) (8.2 years for 2013). EIOPA (2014) reports an average duration of 9.5 years for government and 6.9 years for corporate bonds for 2013.

We scale these durations up to the average value reported in EIOPA (2016, Table 12) for 2015, implying the scaling factor $\hat{w}_{2015} = \frac{9.3}{(6.9w_{\text{corp}}+9.5w_{\text{sov}})/(w_{\text{corp}}+w_{\text{sov}})} \approx 1.09$. To calibrate heterogeneity within the government bond portfolio, we use the distribution of the modified duration of government bonds across countries reported in EIOPA (2016, Table 13) and scale these up to match the average government bond portfolio duration of $9.5 \cdot \hat{w}_{2015} = 10.4$. Similarly, to calibrate heterogeneity within the corporate bond portfolio, we use the distribution of modified durations of corporate bonds across ratings reported in EIOPA (2016, Table 14) and scale these up to match the average corporate bond portfolio duration of $6.9 \cdot \hat{w}_{2015} = 7.5$. The final allocation of bonds across ratings is skewed toward higher-rated assets, consistent with those reported by Assekurata (2016).

Given the duration of individual bonds and the target duration of each asset class, we determine portfolio weights following the methodology in Berdin et al. (2017), which assumes that individual bonds' portfolio weights are an exponential function of their remaining time to maturity, and we correct for potential deviations from the target duration by minimizing the square of the difference between target and actual duration starting with the Berdin et al. (2017)-implied weights.

D.5 Calibration of the Short-Rate Model

Short rate dynamics are given by

$$dr_t = \alpha_r(\theta_r - r_t)dt + \sigma_r dW_t^r, \tag{IA.6}$$

where r_t is the short rate at time t, W_t^r is a standard Brownian motion, $\alpha_r > 0$ is the speed of mean reversion, $\sigma_r > 0$ is the volatility, and θ_r is the level of mean reversion. Under the assumption of arbitrage-free interest rates, Equation (IA.6) specifies the term structure of annually compounded interest rates at time t for maturities τ, $\{r_{f,t,\tau}\}_{\tau \geq 0}$. Following Brigo and Mercurio (2006), the price of a zero-coupon bond at time t with maturity at $t+\tau \geq t$ is

$$(1 + r_{f,t,\tau})^{-\tau} = A(\tau)e\left[-B(\tau)r_t\right], \tag{IA.7}$$

where

$$B(\tau) = \frac{1}{\kappa_r}\left(1 - exp\left[-\kappa_r \tau\right]\right)$$

and

$$A(\tau) = exp\left[(\theta_r - \frac{\sigma_r^2}{2\kappa_r^2})(B(\tau) - \tau) - \frac{\sigma_r^2}{4\kappa_r}B(\tau)\right],$$

and $r_{f,t,\tau}$ is the annually compounded interest rate at time t.

We calibrate the short rate volatility σ_r using a maximum-likelihood estimator based on the monthly Euro OverNight Index Average (EONIA) from December 2000 to November 2015.[8] To calibrate κ_r and θ_r, we additionally use the whole term structure of German

[8]EONIA is the weighted rate for the overnight maturity, calculated by collecting data on unsecured

government bond rates. For this purpose, we use the least squares estimate for κ_r and θ_r comparing the term structure for bonds with a maturity from 1 to 20 years implied by the historical evolution of EONIA and the parameters σ_r, κ_r and θ_r with the actual term structure of German government bond rates. The resulting parameters are $\sigma_r = 0.0052$, $\kappa_r = 0.0813$, $\theta_r = 0.018$. The initial level of the short rate is $r_0 = -0.002$, which is the level of EONIA on December 31, 2015.

D.6 Calibration of the Financial Market Model

Spreads for government and corporate bonds are modeled by Ornstein-Uhlenbeck processes, analogously to the short rate,

$$ds_t^j = k^j(\bar{s}^j - s_t^j)dt + \sigma^j dW_t^j. \tag{IA.8}$$

Therefore, $\{r_{f,t,\tau} + s_t^j\}_{\tau \geq 0}$ is the term structure of bonds of type j at time t.

We calibrate bond spreads and stock and real estate returns based on monthly data from December 2000 to November 2015. Corporate bond rates are given by the effective yield of the AAA/AA/A/BBB-subset of the ICE BofAML US Corporate Master Index (obtained from *FRED St. Louis*), which tracks the performance of U.S. dollar-denominated investment-grade rated corporate debt publicly issued in the U.S. domestic market. To account for the different inflation (expectations) between the EU and U.S., we calculate bond spreads with respect to the yield of U.S. treasuries with a maturity of 10 years (obtained from *FRED St. Louis*).[9] Government bond spreads are calibrated based on the spread relative to German bond rates from December 2000 to November 2015 (obtained from *Thomson Reuters Eikon*), averaged across maturities from 1 to 20 years.

overnight lending in the euro area provided by banks belonging to the EONIA panel. Data source: *ECB Statistical Data Warehouse*.

[9]The results are similar if we take German government bond rates instead.

Table IA.5 describes the sample of bond spreads. Note that we retrieve bond rates (and spreads) for maturities of 1 to 20 years for each government bond, while corporate bond spreads are calculated by comparing the effective yield of the ICE BofAML US Corporate Index to the 10-year yield. We assume that the credit spread is the same across maturities for each bond type and, thus, we calibrate the spread process $\{s_t^j\}_t$ for the average spread across maturities in the case of government bonds. Parameter estimates are based on maximum likelihood and reported in Table IA.5. We assume that coupons are equal to the (government or corporate) bond yield at issuance. Given coupons, we price bonds using the term structure of risk-free rates $r_{f,\tau,t}$ and spreads s_t^j.

Table IA.5. Summary Statistics and Calibration of Bond Spreads.
The table reports summary statistics and maximum-likelihood estimates for the long-term mean ($\bar{s}$), speed of mean reversion (k), and volatility (σ) of the Ornstein-Uhlenbeck process $s^j(t) = k^j(\bar{s}^j - s^j(t))dt + \sigma^j dW^j(t)$ for monthly bond spreads between (a) government bond rates and German government bonds and (b) corporate bond rates and the 10Y U.S. treasury bond rate from December 2000 to November 2015. Government bond rates include observations for 1-year to 20-year maturities, and the calibration is based on the average spread across maturities. Corporate bond spreads are based on the effective yield of ICE BofAML US Corporate Indices and 10-year U.S. treasury rates. *Source: Authors' calculations, Thomson Reuters Eikon (government bonds), FRED St. Louis (corporate bonds).*

Name	# Observations	Mean	Sd	p25	p75	$\bar{s}$	k	σ
French	180	0.003188	0.003176	0.0006895	0.004495	0.003593	0.3574	0.00265
Dutch	180	0.002085	0.001711	0.000651	0.003148	0.002172	0.5086	0.001716
Italian	180	0.01158	0.01214	0.002454	0.016	0.01375	0.2018	0.007465
Spanish	180	0.01086	0.01343	0.000667	0.01692	0.01493	0.1497	0.007071
AAA	180	0.003421	0.006385	-0.0005	0.0057	0.003081	1.09	0.009236
AA	180	0.004504	0.008326	-0.00065	0.0069	0.003427	0.5738	0.008593
A	180	0.009906	0.01017	0.0046	0.01115	0.00832	0.4922	0.009814
BBB	180	0.01847	0.01154	0.0119	0.0215	0.0174	0.5289	0.01164

Stocks and real-estate investments follow geometric Brownian motions (GBMs) that are calibrated to the STOXX Europe 600 index and MSCI Europe real estate index, respectively (retrieved from *Thomson Reuters Eikon*). Table IA.6 reports the descriptive statistics for monthly log-returns. We calibrate the GBM drift and volatility with maximum-likelihood estimates for monthly log-returns, which are also reported in Table IA.6. Stocks pay dividends, and real estate investments pay rents at each year's end. Dividends and rents are assumed to equal the maximum of zero and 50% of the annual return.

Table IA.6. Summary Statistics and Calibration for Stocks and Real Estate.
The table reports summary statistics and maximum-likelihood estimates for geometric Brownian motions for monthly stock and real estate returns from December 2000 to November 2015. Stock returns are based on the STOXX Europe 600 index, and real estate returns are based on the MSCI Europe real estate index. *Source: Authors' calculations, Thomson Reuters Eikon.*

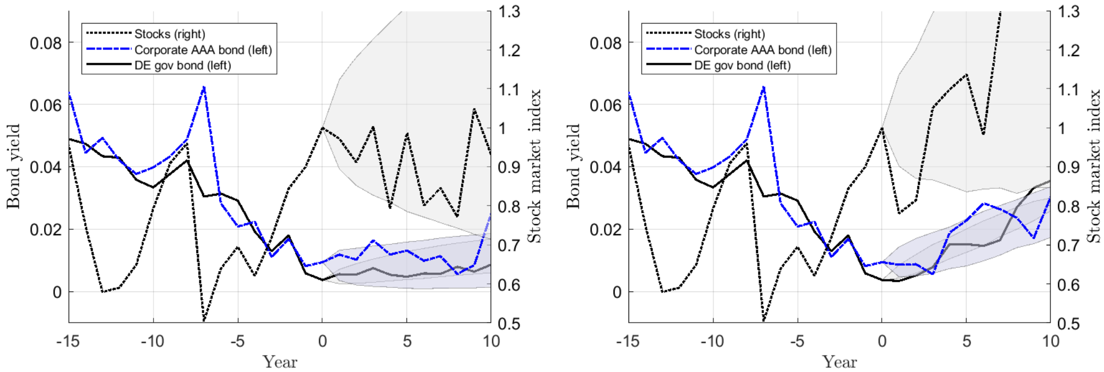
Name	# Observations	Mean	Sd	p25	p75	GBM Drift	GBM Volatility
Stocks	180	0.0001462	0.04879	-0.02109	0.03055	0.01604	0.169
Real Estate	180	0.003853	0.07032	-0.03085	0.04264	0.0759	0.2436

Finally, we correlate all stochastic processes via a Cholesky decomposition of their diffusion terms. Table IA.7 reports the correlation coefficients based on monthly residuals after fitting bond spreads, stock and real estate returns.

Table IA.7. Correlation Matrix for Financial Market Processes.
The table reports the correlation coefficients for monthly residuals from December 2000 to November 2015 of the short rate (EONIA), government bond spreads for France (FR), the Netherlands (NL), Italy (IT), and Spain (ES), corporate bond spreads for AAA-, AA-, A-, and BBB-rated bonds, stocks, and real estate returns, after fitting to the short rate and spreads to Ornstein-Uhlenbeck processes and stocks and real estate returns to geometric Brownian motions.

	EONIA	Spread (FR)	Spread (NL)	Spread (IT)	Spread (ES)	Spread (AAA)	Spread (AA)	Spread (A)	Spread (BBB)	Stocks	Real Estate
EONIA	1	-0.114	-0.133	-0.103	-0.072	-0.073	0.052	0.039	-0.112	0.135	0.274
Spread (FR)	-0.114	1	0.535	0.67	0.629	0.136	0.267	0.284	0.253	-0.174	-0.203
Spread (NL)	-0.133	0.535	1	0.489	0.518	0.278	0.311	0.33	0.368	-0.243	-0.27
Spread (IT)	-0.103	0.67	0.489	1	0.81	0.142	0.277	0.296	0.293	-0.21	-0.196
Spread (ES)	-0.072	0.629	0.518	0.81	1	0.154	0.242	0.252	0.231	-0.147	-0.141
Spread (AAA)	-0.073	0.136	0.278	0.142	0.154	1	0.81	0.773	0.637	-0.095	-0.032
Spread (AA)	0.052	0.267	0.311	0.277	0.242	0.81	1	0.965	0.819	-0.216	-0.08
Spread (A)	0.039	0.284	0.33	0.296	0.252	0.773	0.965	1	0.884	-0.303	-0.179
Spread (BBB)	-0.112	0.253	0.368	0.293	0.231	0.637	0.819	0.884	1	-0.438	-0.342
Stocks	0.135	-0.174	-0.243	-0.21	-0.147	-0.095	-0.216	-0.303	-0.438	1	0.663
Real Estate	0.274	-0.203	-0.27	-0.196	-0.141	-0.032	-0.08	-0.179	-0.342	0.663	1

Figure IA.4. Financial Market Dynamics: Historical and Simulated.
The figures depicts one exemplary simulated path and the 25th / 75th percentiles of simulated 10-year German government bond rates, AAA corporate bond rates, and the European stock market index from year 0 on. Prior to year 0, we show the actual historical evolution, up to year 0, which corresponds to 2015. Figure (a) is based on all simulated paths and Figure (b) is based only on those with the 5% largest average increase in the 10-year German government bond rate.

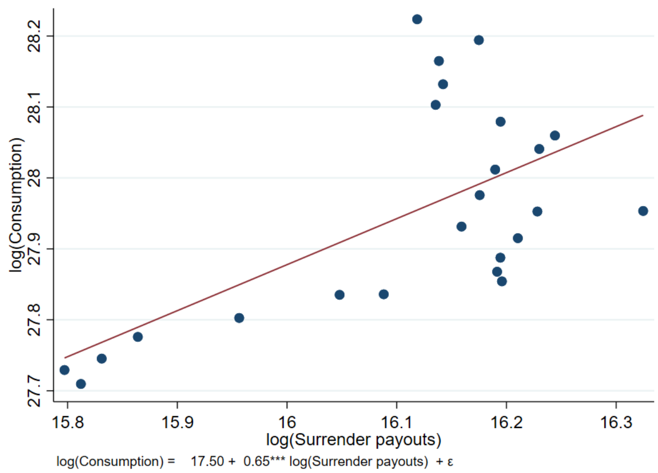


(a) All Paths.

(b) Paths with Rising Interest Rates.

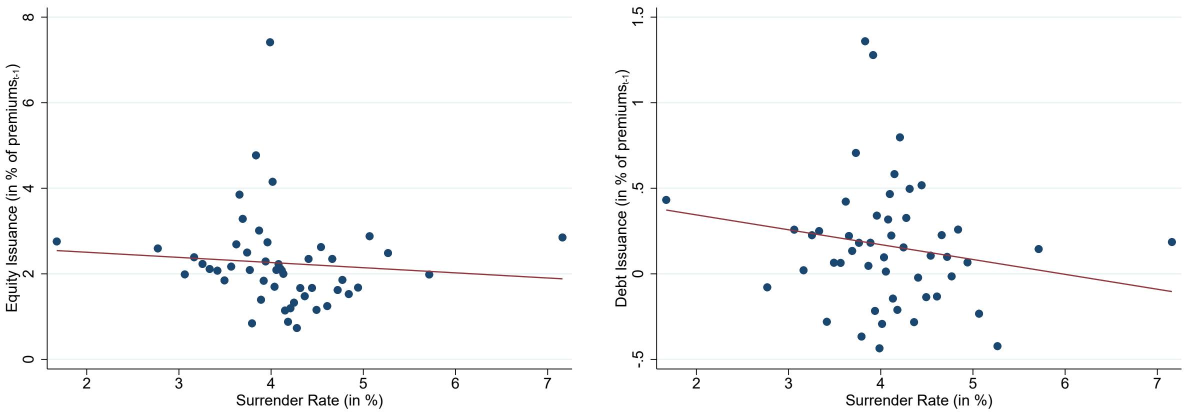
E Surrender Payouts and Consumption

Figure IA.5. Correlation Between Surrender Payouts and Private Consumption.
The figure plots the logarithm of annual aggregate surrender payouts (x-axis) and the logarithm of total private consumption expenditures (y-axis) in Germany from 1996 to 2019 as scatter points. A univariate regression implies that consumption expenditures increase by 0.65% when surrender payouts rise by 1%. *Sources: BaFin (surrender payouts), OECD (private consumption expenditures).*



F Surrenders and Equity and Debt Issuance

Figure IA.6. Correlation Between Surrender Rates and Equity and Debt Issuance.
The figure shows a binned scatter plot of the annual surrender rate (x-axis) and the (a) total equity and (b) total debt issuance (y-axis) of German life insurers at the insurer-by-year level from 2007 to 2019 after absorbing timeinvariant variation using insurer fixed effects. Equity and debt issuance are scaled by lagged gross premiums written. We exclude insurers that never issued (a) equity or (b) debt during this period, with 106 insurers remaining. *Sources: Erstversicherungsstatistik (surrender rates), S&P Capital IQ (equity and debt issuance).*



(a) Equity issuance

(b) Debt issuance

G Additional Simulation Results: Market Value Adjustments

Market value adjustments (MVAs), commonly found in U.S. deferred multiyear annuities (see Internet Appendix A), adjust surrender values for interest rate changes: an increase in interest rates reduces market-value-adjusted surrender values, everything else being equal.

We implement an MVA to examine how it affects surrender rates and asset sales. For this purpose, we use the same initial balance sheet calibration as in the baseline analysis but assume that, starting at $t = 0$, all cohorts' surrender values are subject to an MVA. The market-value-adjusted surrender value at year-begin $t, t \geq 1$, is $sv^h_{t-1,MVA} = (1 - mva^h_{t-1}) \cdot \vartheta \cdot v^h_{t-1}$, where mva^h_{t-1} is the MVA factor. Whereas an MVA may be implemented in various ways, we base the definition of the MVA factor on that most commonly found in the U.S.:

$$mva^h_{t-1} = 1 - \min\left\{ \left(\frac{1 + \tilde{r}^h_{P,t-1}}{1 + \ell + r_{f,t-1,T^h-(t-1)}} \right)^{T-(t-1)}, \vartheta^{-1} \right\}. \tag{IA.9}$$

If $mva^h_{t-1} = 0$, then there is no MVA, and the policyholder receives the cash value less the surrender penalty. The larger mva^h_{t-1}, the smaller is the surrender payout. The minimum operator ensures that the MVA cannot overcompensate the surrender penalty, i.e., policyholders cannot receive more than the contract's cash value. ℓ adjusts the average level of mva^h_{t-1}, accounting for the spread on top of the risk-free rate earned by insurers. A low value of ℓ translates into a low average MVA factor, boosting surrender rates. We use $\ell = 0.0282$, which makes the initial average level of the surrender rate in our model comparable to that in the baseline calibration.

Figure IA.7 shows the distribution of market value adjustment factors mva^h_{t-1} across cohorts and over time. Owing to rising interest rates, adjustment factors increase, depressing adjusted surrender values.

Figure IA.7. Market Value Adjustment Factor.
The figure depicts the market value adjustment factor, as defined in Equation (IA.9). The figure shows the median and 25th/75th percentile for each year.

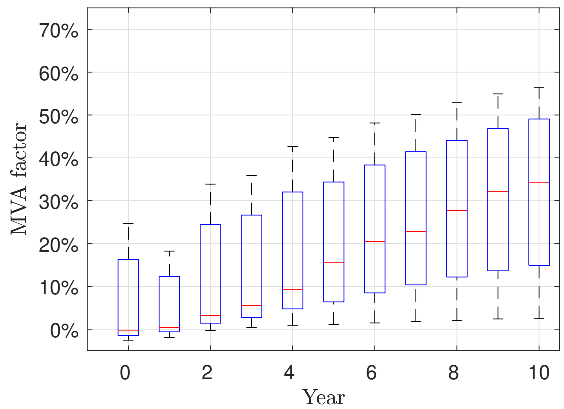
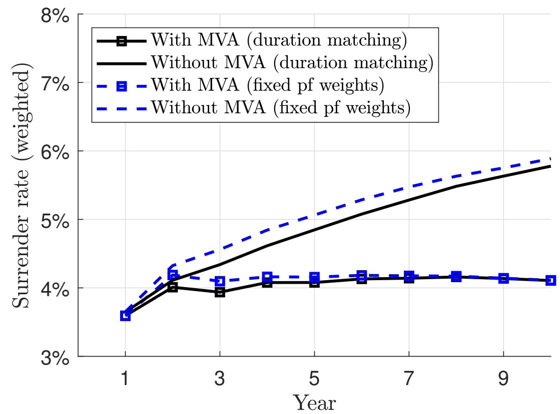


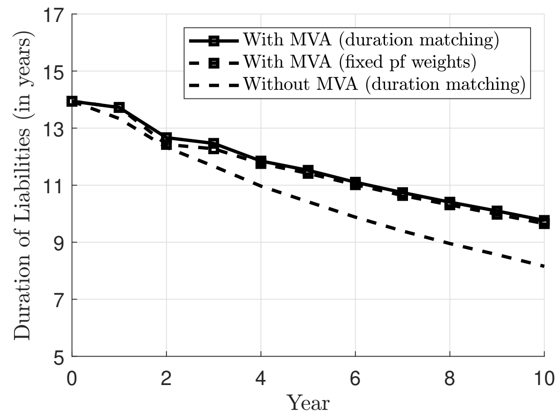
Figure IA.8 compares the surrender rates, durations, asset sales, and price impact in the counterfactual calibration with MVA to that in the baseline calibration.

Figure IA.8. Impact of MVAs.

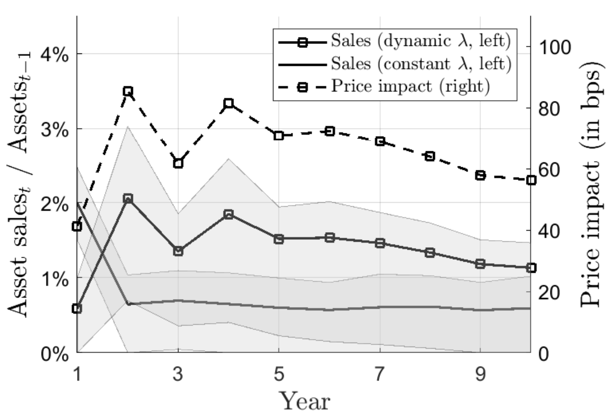
Figures (a) and (b) depict the surrender rates and durations for the baseline calibration without MVAs and the counterfactual calibration with MVAs. Figures (c) and (d) depict the mean and 25th and 75th percentiles of the insurer's asset sales relative to the previous year's total assets for a constant surrender rate λ and a dynamic surrender rate λ (endogenously determined depending on the market environment) as well as the mean price impact per EUR 1 sold with a dynamic surrender rate λ, all with MVAs. We show the results for both the investment strategy with duration matching and that with fixed portfolio weights.



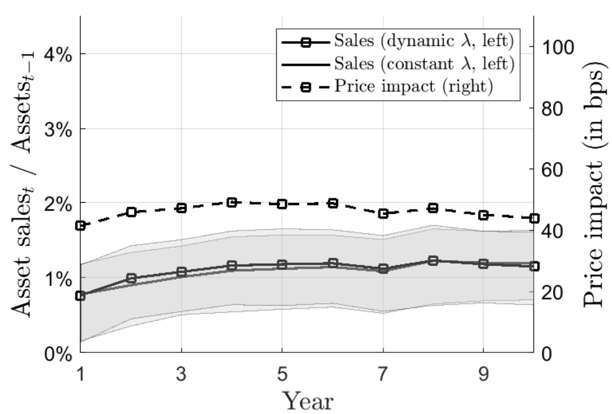
(a) Surrender rates.



(b) Duration of Liabilities.



(c) Asset sales and price impact (duration matching).



(d) Asset sales and price impact (fixed pf weights).

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