Margins as Canaries in the Coal Mine*

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Abstract

Central counterparties (CCPs) manage counterparty risk by requiring margins, pooling default losses, and replacing defaulted clearing members. We show that counterparty replacement is central to how a CCP with limited resources optimally combines these tools. Without replacement, defaults at maturity can trigger cascading failures among clearing members. Replacement can prevent this by transferring positions to stronger counterparties. Margins serve as *canaries in the coal mine*: rather than deterring default, they induce fragile counterparties to default before contract maturity, enabling replacement while risk sharing remains possible. Our model reveals a pecking order of CCP risk management. When counterparty fragility is not widespread, loss sharing among surviving members suffices. When fragility is widespread, it can be optimal for the CCP to use margin calls to identify and replace fragile counterparties, particularly when these are severely impaired. Our findings highlight the complementary nature of CCP risk management tools: margins, loss sharing, and counterparty replacement.

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1 Introduction

Regulatory reforms following the 2008 financial crisis established central counterparties (CCPs) as a cornerstone of global financial infrastructure. As of 2025, about 50% of global over-the-counter (OTC) derivatives trades were centrally cleared, representing more than $500 trillion in notional.[1] The functioning of CCPs is therefore crucial for financial stability. At the same time, our understanding of how central clearing shapes the allocation of risk and the propagation of losses in financial markets remains limited.

In this paper, we develop a tractable model of risk sharing and central clearing that characterizes the three mechanisms through which CCPs manage risk: margins, counterparty replacement, and loss sharing. Margins, i.e., collateral that clearing members must post to cover potential losses, are large in practice: CCPs hold more than $1.5 trillion of margin, with daily flows exceeding $30 billion.[2]

The existing literature has emphasized the role of margins in ring-fencing assets, thereby increasing the resources available upon default and reducing counterparties' incentives to default (Biais et al., 2016). We highlight a second function of margins as *canaries in the coal mine*: a margin call at an interim date can induce fragile counterparties to default before contract maturity, allowing the CCP to replace them. Replacing counterparties early is valuable because the final payment is still uncertain, such that the contracts can be transferred to others while risk sharing remains possible. This reduces the losses imposed on surviving clearing members and can prevent a scenario in which the CCP is overwhelmed by default losses at maturity. From this perspective, margins, counterparty replacement, and loss sharing are therefore complements rather than substitutes.

In our model, risk-averse protection buyers hedge endowment risk by trading centrally cleared derivatives with risk-neutral protection sellers. The terms of the derivative contract as well as margin requirements and loss-sharing arrangements are determined endogenously as the solution to an optimal contracting problem. Following an interim signal about underlying risks, some sellers become fragile as a result of privately observed shocks to their balance sheets, and may then be unable to make their contractual payments at maturity. If sufficiently many fragile sellers default, their losses can exhaust the CCP's ex-post loss-sharing capacity, causing surviving members to fail in turn and, ultimately, the CCP itself.

[1]The clearing share is weighted by notional, includes foreign exchange, interest, and equity-linked contracts, and is adjusted for potential double-counting of inter-dealer trades novated to CCPs. Source: BIS OTC derivatives statistics, available at https://data.bis.org/topics/OTC_DER.

[2]Source: CCP Global (2025) "Public Quantitative Disclosure PQD Quarterly Trends Report 2024 Q4 Data".

Our main result is that the CCP can prevent this scenario by using margin calls to identify and replace fragile counterparties. Fragile sellers cannot meet the interim margin call and default, which allows the CCP to move their positions to counterparties with more resources. By inducing early defaults before contract maturity, margin calls prevent default cascades that would otherwise overwhelm the CCP.

However, margins are not always optimal. When only few clearing members are fragile, loss sharing at the CCP suffices to maintain full insurance: surviving members can absorb the losses of those that default without failing themselves. Because posted margin earns less than sellers' alternative use of the funds, calling it is costly, and the CCP does not use them.

When fragility is more widespread, margins become optimal, either to ring-fence assets or as canaries. Ring-fencing can be valuable because it secures collateral from potential defaulters, reducing deadweight losses when defaults occur at maturity. That is especially useful when surviving sellers' resources are limited. Canary margins instead create value by revealing which counterparties are fragile, allowing their positions to be reallocated to those best able to bear them. The two roles call for different margin levels: a ring-fencing margin only secures a fraction of a defaulter's assets, whereas a canary margin must be high enough that fragile sellers cannot meet it. The canary role therefore dominates when fragile sellers' balance sheets are severely impaired. In this case, a small margin call makes them default and, thus, replacement is cheap. At the same time, the impaired balance sheets of fragile sellers do not offer many assets that could be ring-fenced.

Our model clarifies how the relationship between margins, replacement, and loss sharing changes over a contract's lifetime. At maturity, margins and loss sharing are substitutes: ring-fencing a larger share of defaulters' assets leaves fewer losses to be shared. Replacement is of no use at that point since the cost of taking over a contract is at least as large as the payment due, leaving no scope for risk sharing. Before maturity, the relationship is reversed. At that point, final payments are still uncertain and, thus, counterparties remain willing to provide insurance and replacement preserves risk-sharing opportunities. Margin calls enable it by identifying fragile counterparties, and the prospect of sharing default losses makes replacement more valuable. The three tools are therefore complements rather than substitutes before maturity.

Whether replacement is efficient depends on the resources available in the market and on what is known when it takes place. Specifically, we highlight three determinants. First, replacement requires sufficient heterogeneity among market participants: those taking over defaulted positions must have the resources to bear them. Second, timing matters because of the Hirshleifer (1971) effect: the more information about underlying risks has been revealed, the less risk sharing replacement can provide. Replacing counterparties early is therefore more valuable than replacing them late. Third, replacement is more attractive when fragile sellers are severely impaired, since a smaller margin call then suffices to induce their default.

Our framework yields three main policy implications. First, optimal risk management follows a pecking order. When there are only few fragile sellers, loss sharing suffices and no replacement is needed. It may be optimal to use margins if it is otherwise difficult to seize defaulters' assets. Instead, when there are many fragile sellers, it is optimal to let fragile sellers default on a canary margin call and to replace them. Second, the model explains why clearing member defaults often follow margin calls rather than occurring at contract maturity, as in the 2018 default of the power trader Einar Aas at Nasdaq Clearing. Third, raising margins in response to adverse shocks can be efficient when it facilitates replacement, provided counterparties with sufficient resources are available. Thus, procyclical margins can build resilience rather than amplify systemic risk.

These results have important implications for financial regulation. Current policy debates focus on calibrating margins to cover potential losses while avoiding destabilizing procyclicality. This perspective is incomplete: margins should also be evaluated for their role in enabling counterparty replacement. Rules that cap margin volatility may therefore hamper their value in identifying and replacing fragile clearing members, raising rather than lowering systemic risk. Moreover, our results highlight the importance of CCPs' access to counterparties with ample resources to potentially take over defaulted positions. Ensuring and monitoring such excess resources may thus deserve relatively more attention in current regulatory frameworks.

Related literature. This paper contributes to the literature on financial market architecture and systemic risk (Allen and Gale, 2000; Zawadowski, 2013; Acemoglu et al., 2015) and frictions in derivatives markets (Bolton and Oehmke, 2015; Biais et al., 2021; Allen and Wittwer, 2023). Specifically, we show how replacing counterparties before maturity can prevent the propagation of default losses across clearing members.

Our paper is most closely related to the literature on CCPs as insurers of counterparty risk (Acharya and Bisin, 2014; Biais et al., 2016; Capponi et al., 2022b; Cucic, 2022; Kuong and Maurin, 2023). We add to this literature by studying the role of counterparty replacement and its interaction with loss sharing and margins. Biais et al. (2016) is closest to our analysis. They highlight that margins reduce counterparties' incentives to default, and consider CCPs with unlimited resources. We instead consider a setting in which CCP resources are limited, which generates a role for counterparty replacement: margins are set to induce default rather than to prevent it. Several other studies on CCPs examine the role of netting and the demand for clearing (Duffie and Zhu, 2011; Kubitza et al., 2024; Chebotarev, 2025; Sannino and Zeng, 2026), settlement (Koeppl et al., 2012), and the design of CCP auctions (Vuillemey, 2023; Huang and Zhu, 2024). A number of empirical studies document the impact of central clearing on counterparty risk and financial markets (Loon and Zhong, 2014; Boissel et al., 2017; Bernstein et al., 2019; Vuillemey, 2020) as well as margining practices (Capponi et al., 2022a; Grothe et al., 2023).[3]

Because margin calls allow the CCP to identify fragile counterparties, our analysis also relates to the literature on collateral as a screening device (Bester, 1985; Besanko and Thakor, 1987). That literature studies screening at contract origination, whereas margins in our model screen counterparties after contracts have been entered, which makes the Hirshleifer effect relevant, that is absent in the static setting. Our results also depart from the traditional view of collateral as a means to raise borrowing capacity to finance investment (Holmstrom and Tirole, 1997; Rampini and Viswanathan, 2010; Donaldson et al., 2020). Rather than borrowing capacity, we highlight the use of margins to increase risk-sharing capacity.

2 Model

We develop a simple model of centrally cleared financial contracts used to hedge endowment risk. A natural interpretation of these contracts is centrally cleared derivatives, for example, swaps.

Model overview. There are three dates, $t = 0, 1, 2$. At $t = 0$, protection buyers enter risk-sharing contracts (which we refer to as "derivatives") with protection sellers. The contracts are centrally cleared by a CCP. At $t = 1$, a signal regarding the likelihood that a protection seller will have to make a payment on the derivative contract is observed. At the same time, some protection sellers suffer an adverse shock to their balance sheets. In response, the CCP can require sellers to post margin on their contracts. Sellers with weak balance sheets may default on the interim margin call. If this happens, the CCP can transfer these contracts to other counterparties. At $t = 2$, the derivative contracts pay off, and protection sellers with weak balance sheets may default on their payment obligations.

We now describe each element of the model in detail.

[3] See Menkveld and Vuillemey (2021) for a survey of the literature on central clearing.

Protection buyers and sellers. There is a mass one of protection buyers and a corresponding mass of protection sellers. Protection buyers are identical, with a twice-differentiable concave utility function u over consumption at $t = 2$, satisfying $u' > 0$ and $u'' < 0$. They face endowment risk $\tilde{e}$ at $t = 2$. For simplicity, we assume that $\tilde{e}$ can only take one of two values: $\bar{e} = 1$ with probability $\pi \in (0, 1)$ and $\underline{e} = 0$ with probability $1 - \pi$. The endowment risk $\tilde{e}$ is the same for all protection buyers (i.e., it represents aggregate risk).

Protection buyers can purchase insurance against their endowment risk from risk-neutral protection sellers. Each protection seller is endowed with one unit of an asset (representing their "balance sheet"), whose payoff is realized at $t = 2$. There are two types of assets: low and high quality. Asset quality is unknown at $t = 0$ and privately revealed to protection sellers at $t = 1$, as specified in more detail below.

Interim information. At the beginning of $t = 1$, before margin calls are made and sellers might default, a public signal $\tilde{s}$ about the endowment risk $\tilde{e}$ becomes available. For example, if the risk $\tilde{e}$ stems from the oil price at $t = 2$, $\tilde{s}$ can be interpreted as the oil price at $t = 1$. The signal is correct with probability λ,

$$\lambda = \mathbb{P}(\bar{s} \mid \bar{e}) = \mathbb{P}(\underline{s} \mid \underline{e}). \tag{1}$$

Following Bayes Rule, the probability of a good endowment realization $\bar{e}$ is then updated to

$$\overline{\pi} = \mathbb{P}(\bar{e} \mid \bar{s}) = \frac{\lambda\pi}{\lambda\pi + (1-\lambda)(1-\pi)} \text{ or } \underline{\pi} = \mathbb{P}(\bar{e} \mid \underline{s}) = \frac{(1-\lambda)\pi}{(1-\lambda)\pi + \lambda(1-\pi)}. \tag{2}$$

We assume that $\frac{1}{2} < \lambda < 1$, so that observing $\bar{s}$ is positive news, implying that $\underline{\pi} < \pi < \overline{\pi} < 1$. The larger λ, the more informative the signal. The restriction that $\lambda < 1$ ensures that the signal is not perfectly revealing.

Contracts, margins, and central clearing. Protection buyers and sellers enter a derivative contract at $t = 0$, which is centrally cleared by the CCP. Clearing transforms each bilateral contract between one buyer and one seller into two cleared contracts: one between the seller and the CCP, specifying a transfer T_S, and one between the CCP and the buyer, specifying a transfer T_B. Positive transfers represent payments to the CCP. Negative transfers represent payments made by the CCP.

The contractual transfers T_B and T_S are agreed upon at $t = 0$ and realized at $t = 2$.

These transfers can be made conditional on all public information, including the realizations of the buyers' endowment risk $\tilde{e}$ and the public signal $\tilde{s}$. Because the CCP is a counterparty to every contract, it pools all transfers from buyers, sellers, and (if counterparties default and are replaced at the interim date) outsiders. As a result, the transfer received by protection buyer j does not depend on the transfer made by its original counterparty, but on the transfers made by all clearing members, which determine the resources available to the CCP. Therefore, the transfers T_S and T_B can be interpreted as the total (net) transfers jointly determined by the initial bilateral derivative contracts and pooling of contracts at the CCP (in particular, loss sharing as described below).[4]

The CCP's budget constraint requires that aggregate transfers to protection buyers are matched by aggregate transfers from protection sellers and outside sellers in each state. For example, if no protection seller defaults, the CCP's budget constraint is given by

$$T_B(\tilde{s}, \tilde{e}) + T_S(\tilde{s}, \tilde{e}) = 0 \quad \forall \tilde{s}, \tilde{e}. \tag{3}$$

Margins. In addition to the transfers T_S and T_B, the CCP can require that protection sellers post some of their assets as margin at $t = 1$. The margin call can be made contingent on all public information at $t = 1$ (i.e., the signal $\tilde{s}$). We denote the amount of high-quality assets to be deposited in the margin account by $\alpha(s)$, $s \in \{\underline{s}, \bar{s}\}$. Because of their lower value, a larger amount of low-quality assets is required to satisfy the margin call.

A margin call serves to ring-fence a fraction of sellers' assets. Defaulting sellers lose all posted margin, which is seized by the CCP and distributed among surviving clearing members. However, posting margin is costly: assets in the margin account earn a lower return, resulting in a per-unit opportunity cost of $1 - k \in (0, 1)$. Therefore, the CCP makes a margin call only if doing so increases the (allocation of the) CCP's resources after accounting for the opportunity cost of margins.

Defaults. Equilibrium payments by protection sellers are subject to default. Default occurs when protection sellers are either unable to honor their payment obligations at $t = 2$ or unable to fulfill their margin requirements at $t = 1$. Defaults imply the loss of clearing membership and the termination of any existing derivative contracts. The CCP can use the margin posted by defaulted sellers to make payments to other sellers

[4]For example, protection buyers might trade a credit default swap (CDS) with protection sellers, which provides an insurance payment if the underlying defaults (state $\underline{e}$). In turn, if the underlying does not default, then protection buyers pay an insurance premium (state $\bar{e}$). These payments are conditional on the original's counterparty survival. After centrally clearing the contract, transfers T^B and T^S to the CCP are independent of the original counterparty's survival. However, the transfers now include the contributions to cover the CCP's overall default loss in addition to the CDS insurance premium or payment.

or buyers. In contrast, its access to defaulters' balance sheets in excess of the margin account is limited. Specifically, the CCP can recover only a fraction $\rho \in (0,1)$ of non-collateralized assets from defaulters. The lower ρ, the lower is the CCP's ability to enforce its claim against defaulters' non-collateralized assets. The remaining fraction $1-\rho$ of uncollateralized assets are lost, creating a deadweight cost of default.

While protection sellers are ex-ante identical, the value of their assets is subject to an idiosyncratic shock at $t=1$. We denote by $R(\tilde{s}) > 0$ the payoff of high-quality assets conditional on the signal $\tilde{s}$. For simplicity, we assume that after a positive signal $\tilde{s} = \bar{s}$, seller resources $R(\bar{s})$ are sufficient to cover contractual derivative payments for all sellers. There is therefore no default risk after a good signal. In contrast, after a negative signal $\tilde{s} = \underline{s}$, there is counterparty risk. Specifically, we assume that a mass γ of sellers becomes fragile because their assets turn out to be of low quality.[5] This fragility shock reduces the payoff of fragile sellers' assets from $R(\underline{s})$ to $\phi R(\underline{s})$, where $\phi \in [0,1)$. Assets of "safe" sellers, which are not affected by the fragility shock, remain high-quality and continue to pay $R(\underline{s})$. We assume that $R(\underline{s}) > \pi$, which ensures that safe sellers have sufficient resources to write a frictionless full insurance contract. The realization of the fragility shock is privately known to sellers but cannot be observed by the CCP. It is therefore not feasible to write a contract conditional on seller type.

Contract replacement, outsiders, and loss sharing. When a protection seller defaults on the margin call at $t=1$, the CCP terminates the contract and proceeds to sell the position of the defaulted clearing member to outsiders, who then replace the defaulted seller as counterparties. Outsiders are not present at $t=0$ and can therefore be interpreted as clearing members (or other market participants) who do not specialize in trading derivative contracts on $\tilde{e}$. To reflect this, we allow for the possibility that transferring defaulted contracts to outside sellers generates a (deadweight) replacement cost of $C \geq 0$ per contract.[6]

Similar to the initial protection sellers, outside sellers are risk-neutral and have assets that pay $R_O > 0$ at $t=2$.[7] R_O can be interpreted as reflecting aggregate financial condi-

[5]Therefore, the informativeness of the signal λ reflects the correlation between seller fragility and information about the derivative (e.g., the price of the underlying). If λ is high, sellers are more fragile when the adverse endowment realization $\underline{e}$ becomes more likely. Correlation between counterparty fragility and exposure to this counterparty is commonly referred to as wrong-way risk.

[6]In practice, this cost includes the cost of documentation and accreditation for clearing membership and other replacement costs that arise when transferring defaulted derivative contracts to a new counterparty.

[7]In Appendix B, we consider an alternative setting without outside sellers. In this extension, all protection sellers are present from $t=0$. At $t=1$, sellers learn whether they are safe, mediocre, or fragile (i.e., there are now three rather than two seller types at the interim date). Safe sellers then take the role of outsiders and take over the defaulted contracts of fragile sellers.

tions at $t = 1$: the smaller R_O, the lower the CCP's ability to find suitable counterparties to take over defaulted contracts. Because defaults at $t = 1$ occur after a negative signal realization $\underline{s}$, outsiders generally need to be compensated for taking on the expected liability associated with the open positions of a defaulted clearing member. Therefore, even though defaulted counterparties can be replaced at $t = 1$, the CCP incurs a default loss that must be borne by the surviving clearing members.

Equilibrium We consider equilibria that maximize the expected utility of protection buyers, subject to the CCP's budget constraints and sellers' resource and participation constraints. This is a natural outcome when the CCP maximizes the welfare of its members (e.g., because the CCP is member-owned) and protection sellers are competitive.[8]

3 First Best: No Defaults

In this section, we consider the benchmark case in which protection seller resources are abundant. In this case, there are no defaults and the first-best outcome is achieved. This case serves as a benchmark for identifying inefficiencies that arise from limited resources and defaults.

Because there are no defaults, no (costly) margins are used. There is also no need to transfer contracts to outside sellers at $t = 1$. The derivative contract specifies transfers from buyers and sellers to the CCP, T_B and T_S, to maximize buyers' expected utility,

$$\mathbb{E}[u(\tilde{e} - T_B(\tilde{s}, \tilde{e}))], \tag{4}$$

subject to the CCP's budget constraint (3) and the sellers' participation constraint. Given no defaults and no margins, the sellers' expected payoff upon entering the contract at $t = 0$ is $\mathbb{E}[\tilde{\phi} R(\tilde{s})] - \mathbb{E}[T_S(\tilde{s}, \tilde{e})]$, where $\tilde{\phi}$ denotes the fragility shock (equal to $\tilde{\phi} = \phi$ for fragile sellers after observing $\underline{s}$, and $\tilde{\phi} = 1$ otherwise) and $R(\tilde{s})$ is the asset payoff (equal to either $R(\underline{s})$ or $R(\bar{s})$). If sellers do not enter the contract, their expected payoff equals $\mathbb{E}[\tilde{\phi} R(\tilde{s})]$. The protection sellers' participation constraint is therefore

$$\mathbb{E}[T_S(\tilde{s}, \tilde{e})] \leq 0. \tag{5}$$

[8]In our model, the CCP's objective does not conflict with the planner's objective. It is, therefore, not necessary to model the CCP's balance sheet beyond its margin account. Several prior studies analyze conflicts of interest between CCPs and protection buyers (see Huang, 2018; Kuong and Maurin, 2023). It would be an interesting avenue for future research to explore the interaction of such conflicts with contract replacement.

Proposition 1 (First-best contract)**.** *When seller resources are abundant, the optimal contract provides full insurance to protection buyers, is actuarially fair, and does not depend on the signal. No margins are used, and outside sellers remain inactive. The transfers from buyers to the CCP are given by*

$$T_B(\tilde{s}, \bar{e}) = 1 - \pi \text{ and } T_B(\tilde{s}, \underline{e}) = -\pi, \tag{6}$$

and those from sellers to the CCP by

$$T_S(\tilde{s}, \bar{e}) = -(1 - \pi) \text{ and } T_S(\tilde{s}, \underline{e}) = \pi. \tag{7}$$

The first-best contract implements full insurance through transfers that effectively move resources from buyers to sellers in the good state $\bar{e}$ and from sellers to buyers in the bad state $\underline{e}$. Transfers are such that buyers are fully insured: buyer consumption is constant across states and equal to the expected endowment π. Note that the CCP is superfluous in this case. The CCP's sole role is to intermediate default-free payments between buyers and sellers. Since all parties can meet their obligations, there is no counterparty risk.

4 Central Clearing Without Replacement

We now consider the case in which protection sellers have limited resources. We first examine contracts with default risk but without replacement of counterparties. We begin by characterizing the transfers and margins after a positive signal at $t = 1$.

Proposition 2 (Full insurance and no margins after a positive signal)**.** *Conditional on a good signal, $\tilde{s} = \bar{s}$, the optimal contract requires no margins and provides full insurance to protection buyers: buyer consumption $\tilde{e} - T_B(\tilde{e}, \bar{s})$ is constant across all endowment realizations $\tilde{e}$.*

Conditional on a good signal, resources are abundant and the optimal contract therefore transfers the full endowment risk from buyers to sellers. Consequently, buyers' final consumption does not depend on the endowment realization. Given the absence of defaults after a good signal, it is optimal not to use margins because they would impose unnecessary costs (since $k < 1$). To simplify notation, in what follows we therefore denote by α the margin requirement after a bad signal (i.e., $\alpha := \alpha(\underline{s})$).[9]

[9] While this section focuses on central clearing without replacement, Proposition 2 applies more generally to the cases with and without replacement of fragile sellers.

After a bad signal, protection sellers may default. We first consider contracts for which defaults do not occur along the equilibrium path.

No defaults. The sellers' participation constraint requires that expected transfers compensate for the opportunity cost of posting margin:

$$0 \leq -\mathbb{P}(\underline{s})\alpha(1-k)R(\underline{s}) - \mathbb{E}[T_S]. \tag{8}$$

To prevent default, the sellers' resource constraints must be satisfied. The available resources consist of the sellers' initial assets that are not posted as margin as well as the assets in their margin account. A margin call requires sellers to post α units of high-quality assets as margin. Safe sellers can post high-quality assets to satisfy the margin call. Fragile sellers, who do not have high-quality assets on their balance sheet, need to post $\frac{\alpha R}{\phi R} = \frac{\alpha}{\phi} > \alpha$ units of low-quality assets to satisfy the margin call. Sellers of type $j \in \{S, F\}$ do not default at $t = 1$ if their available resources are sufficient to cover the margin call:

$$\alpha R(\underline{s}) \leq \phi_j R(\underline{s}) =: \bar{t}_{j,1}, \tag{9}$$

where $\phi_j \in \{\phi, 1\}$ is the fragility shock suffered by sellers of type j.[10]

Sellers of type j do not default at $t = 2$ if their available resources $\bar{t}_{j,2}$ are sufficient to make the contractual transfer $T_S(\underline{s}, \tilde{e})$:

$$T_S(\underline{s}, \tilde{e}) \leq \underbrace{\left(1 - \frac{\alpha}{\phi_j}\right)\phi_j R(\underline{s})}_{\text{Uncollateralized assets}} + \underbrace{\alpha k R(\underline{s})}_{\text{Margin account}} =: \bar{t}_{j,2}. \tag{10}$$

If fragile sellers are sufficiently unconstrained, the optimal contract implements unconditional full insurance at actuarially fair prices:

Proposition 3 (Full insurance without defaults)**.** *The first-best contract described in Proposition 1 can be implemented if and only if the fragility shock is not too severe:*

$$\phi R(\underline{s}) \geq \pi. \tag{11}$$

In the case of Proposition 3, the resources of fragile sellers are sufficiently large to ensure that they do not default. Therefore, there is sufficient risk-sharing capacity to im-

[10] In our model, defaults are driven by limited resources. In an extension, we show that the results naturally extend to a setting with strategic defaults, in which defaults are driven by default incentives.

plement the first-best full insurance contract. In the following, we focus on situations in which fragile sellers are constrained, such that the first-best contract cannot be implemented.

Assumption 1. *Fragile sellers' resources are sufficiently limited such that the first-best contract is not feasible: $\phi R(\underline{s}) < \pi$.*

When the first-best contract is not feasible, there are two options. The first is to write a partial insurance contract that prevents default. The second is to write a full insurance contract that involves default in some states. We now consider each in turn.

Partial insurance without defaults. When the transfers under the first-best contract exceed fragile sellers' resources, one option is to prevent default by reducing the insurance payment made by sellers. To ease notation, in what follows we denote by $u(\tilde{s}, \tilde{e}) = u(\tilde{e} - T_B(\tilde{s}, \tilde{e}))$ the buyer's utility in state $(\tilde{s}, \tilde{e})$.

Proposition 4 (Partial insurance without defaults)**.** *Under Assumption (1), the optimal contract when fragile sellers do not default provides partial insurance to buyers, with full insurance except in the worst state $(\underline{s}, \underline{e})$:*

$$u(\underline{s}, \underline{e}) < u(\underline{s}, \overline{e}) = u(\overline{s}, \underline{e}) = u(\overline{s}, \overline{e}). \tag{12}$$

Fragile sellers' resource constraint (10) is binding in state $(\underline{s}, \underline{e})$ at $t = 2$. No margin is used, and the contract is actuarially fairly priced.

The contract in Proposition 4 prevents fragile sellers from defaulting by reducing the transfers sellers are required to make. This reduction in seller transfers implies that the contract provides only partial insurance to buyers. Margins are not used because, through their opportunity cost, they reduce the resources available to sellers who do not default in equilibrium.

Full insurance with defaults. Instead of preventing fragile sellers from defaulting, it can be optimal to allow fragile sellers to default. Due to the pooling of contracts within the CCP, safe sellers step in to make the contractual payments of defaulted fragile sellers. This loss-sharing arrangement is priced into the contract ex ante via the sellers' participation constraint. In addition, the CCP can use assets seized from defaulting sellers to make contractual payments. While the CCP can seize all margin posted by defaulted sellers, due to enforcement frictions it can seize only a fraction ρ of defaulting sellers' uncollateralized assets. Therefore, when fragile sellers default in state $(\underline{s}, \underline{e})$ at $t = 2$, the CCP's budget constraint in this state is

$$T_B(\underline{s},\underline{e})+\underbrace{(1-\gamma)T_S(\underline{s},\underline{e})}_{\text{Surviving (safe) sellers}}+\underbrace{\gamma\left[\alpha kR(\underline{s})+\rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right]}_{\text{Defaulting (fragile) sellers}}=0. \tag{13}$$

In all other states, the CCP's budget constraint is given by Equation (3), as before.

When fragile sellers default, they lose all their assets. Therefore, the sellers' participation constraint becomes

$$\begin{aligned}\mathbb{E}[\tilde{\phi}R(\tilde{s})]\leq\mathbb{P}(\bar{s})\quad&\underbrace{\left(R(\bar{s})-\mathbb{E}[T_S\mid\bar{s}]\right)}_{\text{No defaults after a positive signal}}\\&+(1-\gamma)\mathbb{P}(\underline{s})\underbrace{\left(R(\underline{s})-\mathbb{E}[T_S\mid\underline{s}]-(1-k)\alpha R(\underline{s})\right)}_{\text{Safe sellers: no defaults}}\\&+\gamma\mathbb{P}(\underline{s})\underbrace{\left(\mathbb{P}(\bar{e}\mid\underline{s})(\phi R(\underline{s})-T_S(\underline{s},\bar{e})-(1-k)\alpha R(\underline{s}))+\mathbb{P}(\underline{e}\mid\underline{s})\cdot 0\right)}_{\text{Fragile sellers: defaults in state }(\underline{s},\underline{e})},\end{aligned} \tag{14}$$

which, after rearranging, yields

$$0\leq-\mathbb{E}[T_S]-\mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s})+\gamma\mathbb{P}(\underline{s},\underline{e})\left(T_S(\underline{s},\underline{e})+((1-k)\alpha-\phi)R(\underline{s})\right). \tag{15}$$

Equation (15) shows that sellers require compensation for the opportunity cost of posting margin, offset by the benefit of defaulting. This benefit equals the difference between the payment by safe sellers, $T_S+(1-k)\alpha R(\underline{s})$, and the payment by defaulted sellers, $\phi R(\underline{s})$. The resource constraint at $t=1$ remains the same as before and is given by Equation (9).

Proposition 5 (Full insurance with defaults at $t=2$)**.** *Under Assumption 1 and if the mass of fragile sellers is sufficiently small,*

$$\gamma\leq\frac{R(\underline{s})-\pi-\mathbb{P}(\bar{s})(1-k)\alpha^{NR}R(\underline{s})}{R(\underline{s})\left(1-\left[\mathbb{P}(\underline{s},\underline{e})(1-\rho)+\rho\right]\phi-(1-\rho)\left[1-\mathbb{P}(\underline{s},\underline{e})\right]\alpha^{NR}\right)}=:\overline{\gamma}^{NR}(\alpha^{NR}), \tag{16}$$

the optimal contract with defaults of fragile sellers in state $(\underline{s},\underline{e})$ provides full insurance to buyers.

The optimal margin α^{NR} is equal to zero if

$$\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho)<\mathbb{P}(\underline{s})(1-k), \tag{17}$$

and equal to ϕ otherwise.[11] *The markup paid by buyers compensates for the opportunity costs of*

[11] If $\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho)=\mathbb{P}(\underline{s})(1-k)$, all $\alpha\in[0,\phi]$ yield the same markup. We select $\alpha^{NR}=\phi$ without loss

posting margins and for the deadweight cost of defaults:

$$m^{NR} = \mathbb{P}(\underline{s})(1-k)\alpha^{NR}R(\underline{s}) + \gamma\mathbb{P}(\underline{s},\underline{e})(1-\rho)\left(\phi - \alpha^{NR}\right)R(\underline{s}). \tag{18}$$

No sellers default at $t = 1$.

Condition (17) captures the trade-off in using margins, namely the opportunity cost of posting margin against the reduction in deadweight default costs. A larger margin ring-fences more of fragile sellers' assets, reducing the fraction of their resources that is lost when they default at $t = 2$. At the same time, posting margin imposes an opportunity cost on all sellers after a negative signal. If Condition (17) holds, the opportunity cost dominates and no margin is used. Otherwise, the default-cost benefit dominates, and it is optimal to set the largest margin that does not induce fragile sellers to default at $t = 1$, $\alpha^{NR} = \phi$.

Proposition 5 shows that, even if fragile sellers default at $t = 2$, safe sellers can still provide full insurance to buyers. This is possible if safe sellers have sufficient resources, which is the case if the mass of defaulting sellers is not too large (Condition (16)). If the mass of defaulting sellers is too large, safe sellers' balance sheets are not sufficient to absorb the default losses in state $(\underline{s},\underline{e})$. If this were the case, the contract in Proposition 5 would lead to contagious defaults: Defaults by fragile sellers would trigger a second round of defaults by safe sellers, as the loss sharing required to implement full insurance would exceed safe sellers' resources.

Partial insurance with defaults. In the following, we assume that the mass of fragile sellers is large, such that safe sellers cannot absorb all losses from defaults of fragile sellers under the full-insurance contract. In this situation, writing the full-insurance contract would lead to cascading defaults: the defaults of fragile sellers would overburden the safe sellers, causing them to default as well. The CCP would fail.

Assumption 2. *Assume that $\gamma > \overline{\gamma}^{NR}(\alpha^{NR})$, such that full insurance with the optimal margin α^{NR} is not resource compatible without replacement of fragile sellers.*

Proposition 6 (Partial insurance with defaults at $t = 2$)**.** *Under Assumption (2), the optimal contract when fragile sellers default in state $(\underline{s},\underline{e})$ provides partial insurance to buyers. The*

of generality.

contract is not actuarially fair but entails a markup paid by buyers to sellers of

$$m = \underbrace{\mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s})}_{\textit{Opportunity cost of margins}} + \underbrace{\gamma\mathbb{P}(\underline{s},\underline{e})\left(\phi-\alpha-\rho\left(\phi-\alpha\right)\right)R(\underline{s})}_{\textit{Net deadweight loss from defaults}}. \tag{19}$$

If margins are used as part of the optimal contract, the margin call is determined by the first-order condition

$$\begin{aligned} u'(\underline{s},\bar{e})\mathbb{P}(\underline{s})(1-k) =& u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho) \\ &+ (u'(\underline{s},\underline{e}) - u'(\underline{s},\bar{e}))\mathbb{P}(\underline{s},\underline{e})(\gamma(k-\rho)-(1-\gamma)(1-k)). \end{aligned} \tag{20}$$

Fragile sellers do not default at $t=1$ if and only if $\alpha \leq \phi$.

The optimal margin requirement determined by Equation (20) trades off the opportunity cost of posting margin (left-hand side) with the benefits of margins (right-hand side). Margins are potentially beneficial for two reasons. First, they reduce the deadweight cost of defaults by ring-fencing assets early on. Specifically, a one-unit increase in α reduces the deadweight-cost component in the markup, $\phi-\alpha-\rho(\phi-\alpha)$, by $1-\rho$. Second, margins can increase the CCP's available resources after a negative signal. On the one hand, a larger margin requirement increases the recoverable resources from fragile sellers by $\gamma(k-\rho)$ per unit, which is positive if the return on margins exceeds the CCP's recovery rate on defaulters' assets, $k>\rho$. On the other hand, a larger margin reduces the available resources provided by safe sellers by $(1-\gamma)(1-k)$ (see Equation (10)). Margins increase resources in state $(\underline{s},\underline{e})$ if the first effect dominates. This is the case if there are sufficiently many fragile sellers, $\gamma > \frac{1-k}{1-\rho}$.

The following proposition compares the two partial insurance contracts analyzed above. These contracts differ in whether fragile sellers default at $t=2$. We show that it is optimal to let fragile sellers default if they are very fragile (i.e., when ϕ is low, making their assets severely impaired).

Proposition 7 (Partial insurance with and without defaults)**.** *The optimal contract with defaults of fragile sellers at $t=2$ provides strictly more expected utility to buyers than that without defaults when fragile sellers are sufficiently impaired, $\phi < \phi^*$, where $\phi^* > 0$.*

5 Central Clearing with Replacement

This section presents the main results of our analysis. We characterize contracts in which the CCP replaces fragile sellers at the interim date $t=1$. Since the CCP cannot distinguish between safe and fragile sellers, it must induce sellers to reveal their type. This is accomplished through a margin call at $t = 1$ that causes fragile sellers (but not safe sellers) to default on their contracts. The defaulting sellers are then replaced by outsiders, who take over the defaulted contracts for a fee.

Analogous to defaults at $t = 2$, the CCP holds a claim on the assets of sellers that default at $t = 1$. This claim equals the expected transfer $\mathbb{E}[T_S \mid \underline{s}]$ (i.e., the market value of the contract at $t = 1$). We assume that, to satisfy its claim, the CCP can seize a fraction ρ of a defaulted seller's uncollateralized assets, leading to a payoff of $\min\{\rho\phi R(\underline{s}), \mathbb{E}[T_S \mid \underline{s}]\}$. In what follows, we assume that ϕ is sufficiently low such that $\rho\phi R(\underline{s}) \leq \mathbb{E}[T_S \mid \underline{s}]$, which implies that the CCP incurs a default loss. The remaining fraction $1 - \rho$ of the defaulting seller's assets is lost, resulting in a deadweight default cost of $(1 - \rho)\phi R(\underline{s})$ at $t = 1$.

Contracts of defaulted sellers are transferred to outsiders. This means that outsiders take over the contractual payment obligation T_S in return for a fee of p. In addition, counterparty replacement generates an exogenous replacement cost C.

Including net transfers made by outsiders and replacement costs, the CCP's budget constraint following a bad interim signal and the default and replacement of fragile sellers becomes

$$T_B + (1-\gamma)T_S + \underbrace{\gamma\rho\phi R(\underline{s})}_{\substack{\text{Resources from}\\ \text{defaulters}}} + \underbrace{\gamma T_S - \gamma(p+C)}_{\substack{\text{Additional resources}\\ \text{from replacement}}} = 0. \tag{21}$$

Outsiders are willing to enter the contract if the payment they receive exceeds the expected transfer,

$$p \geq \mathbb{E}[T_S \mid \underline{s}]. \tag{22}$$

In equilibrium, this participation constraint binds, so that outsiders receive a fee of $p = \mathbb{E}[T_S \mid \underline{s}]$.

Protection sellers' ex-ante participation constraint takes into account the possibility of default at $t = 1$, which means that, with probability $\mathbb{P}(\underline{s})\gamma$, no transfers are made but sellers' assets are lost:

$$0 \leq -\mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\left(\mathbb{E}[T_S \mid \underline{s}] - \phi R(\underline{s})\right) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s}). \tag{23}$$

This constraint shows that, in addition to expected transfer payments, sellers are compensated for losing their assets upon default at $t = 1$, $\mathbb{P}(\underline{s})\gamma\phi R(\underline{s})$, and for the expected opportunity cost of posting margin, $\mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s})$.

5.1 Full Insurance

Using the CCP's budget constraint and the participation constraints for protection sellers and outsiders, we now derive the transfers for contracts that fully insure the buyers' endowment risk.

Proposition 8 (Full insurance transfers with replacement of fragile counterparties)**.** *The transfers of a full insurance contract with default by fragile sellers at $t = 1$ are given by*

$$T_S(\bar{s}, \bar{e}) = -(1 - \pi) - m \tag{24}$$

$$T_S(\bar{s}, \underline{e}) = \pi - m \tag{25}$$

$$T_S(\underline{s}, \bar{e}) = -(1 - \pi) - m + L^{CCP} \tag{26}$$

$$T_S(\underline{s}, \underline{e}) = \pi - m + L^{CCP}. \tag{27}$$

The contract is not actuarially fair but entails a markup paid by buyers to sellers of

$$m := \mathbb{E}[T_B] = \mathbb{P}(\underline{s})(\gamma C + \gamma(1 - \rho)\phi R(\underline{s}) + (1 - \gamma)(1 - k)\alpha R(\underline{s})), \tag{28}$$

and the CCP's default loss after the bad signal is given by

$$L^{CCP} := \gamma(p + C - \rho\phi R(\underline{s})) = \gamma \frac{\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})}{1 - \gamma}. \tag{29}$$

Proposition 8 shows that, under full insurance with interim default and replacement, the contractual transfers for sellers can be decomposed into three components. The first component is a frictionless insurance contract against the endowment risk $\tilde{e}$, where sellers pay π or receive $1 - \pi$ depending on the endowment realization. The second component is a constant markup m that buyers pay to sellers. The markup compensates sellers for the deadweight replacement cost C, the deadweight default cost $(1 - \rho)\phi R(\underline{s})$ per defaulted seller, and the opportunity cost of posting margin $(1 - k)\alpha R(\underline{s})$ per surviving seller. The third component is a loss-sharing contribution that sellers make to cover the CCP's default loss L^{CCP} after a bad signal realization.

The CCP's default loss equals the cost of replacing defaulted sellers, which comprises the payment to outsiders p and the deadweight replacement cost C, net of the assets recovered from defaulted sellers by the CCP, $\rho\phi R(\underline{s})$. Equation (29) shows that the default loss is increasing in the change in the contract's market value from $t = 0$ to $t = 1$ and, in particular, in the change in the endowment probability distribution: $\pi - \underline{\pi}$. The net transfers made by outsiders that take over defaulted contracts are equal to $T_S - p$, which corresponds to a frictionless insurance contract against $\tilde{e}$ conditional on a bad signal, with a net transfer of $\underline{\pi}$ or $-(1-\underline{\pi})$. Therefore, the more informative the interim signal, the less additional risk sharing is provided by outsiders, which, holding the ex-ante markup fixed, raises the default loss. Equation (29) also shows that the default loss is smaller when the CCP can recover more assets from defaulted sellers (i.e., ρ is larger, assuming that fragile sellers are left with some assets after the fragility shock, $\phi > 0$).

The full insurance contract described above requires fragile sellers to default at $t = 1$, since replacement is otherwise impossible. The interim margin call is crucial for achieving this: it induces fragile (but not safe) sellers to default by setting margin requirements that only fragile sellers cannot meet, enabling the CCP to replace fragile counterparties. The margin therefore serves as a canary in the coal mine that reveals hidden fragility among clearing members before contract maturity when replacement is still possible.

Fragile sellers default at $t = 1$ if the margin requirement exceeds their resources, as described by Equation (9).[12] Therefore, fragile sellers default at $t = 1$ when the fraction of assets to be posted as margin exceeds the fragility shock:

$$\alpha > \phi. \tag{30}$$

Because posting margin entails an opportunity cost, it is optimal to set the lowest margin that induces the sellers to default at $t = 1$. Therefore, it is optimal to set the margin to just exceed ϕ:

$$\alpha^* := \inf\{\alpha : \alpha > \phi\}. \tag{31}$$

Note that the role of margins differs depending on whether defaults occur at $t = 1$ or $t = 2$. As shown in the previous section, when defaults occur at $t = 2$, margins reduce the deadweight cost of default by ring-fencing assets for the CCP. However, this mechanism does not apply when defaults occur at $t = 1$, since defaults occur before margin is posted. Instead, the role of margins at $t = 1$ is to induce fragile sellers to default, thereby enabling counterparty replacement.

The optimal transfers are chosen to provide full insurance to buyers, anticipating the

[12] Fragile sellers might want to avoid the margin call by selling their positions to outsiders after a negative signal but before the margin call at $t = 1$. We assume that the margin requirement applies to all sellers with an open position, such that outsiders acquiring the contract must post the same margin. Fragile sellers, however, cannot offload their positions to outsiders when their assets are not sufficient to compensate outsiders: $\phi R(\underline{s}) < \mathbb{E}[T_S \mid \underline{s}] + \alpha(1-k)R(\underline{s})$. Safe sellers, in contrast, do not gain from trading with outsiders before the margin call because the margin cost they avoid is fully reflected in the price required by outsiders.

replacement of fragile sellers via a margin call of α^* at $t = 1$.

Proposition 9 (Full insurance with replacement of fragile counterparties)**.** *The optimal contract with default and replacement of fragile sellers at $t = 1$ provides full insurance if*

$$\gamma \leq \overline{\gamma}^R. \tag{32}$$

It is $\overline{\gamma}^R \in (0, 1)$ if $\phi < \phi^R$, with $\phi^R > 0$.

The optimal margin, which incentivizes fragile sellers to default at $t = 1$, is in the limit given by

$$\alpha = \alpha^* = \phi. \tag{33}$$

The outsiders' resources are sufficient if, and only if, $R_O \geq \underline{\pi}$. This condition is satisfied if $R_O \geq \pi$.

Proposition 9 demonstrates that when the economy contains a sufficiently small proportion of sellers ($\gamma \leq \overline{\gamma}^R$), the full-insurance contract with replacement and defaults at $t = 1$ is optimal and resource-compatible for sellers.

Because the contract's present value changes following the interim signal, defaults generate losses for the CCP, which are absorbed by safe sellers. If the mass of fragile sellers were too large ($\gamma > \overline{\gamma}^R$), safe sellers would lack the capacity to absorb these default losses, rendering the full-insurance contract resource-incompatible.

Outsiders can fulfill their contractual payments if their resources exceed the insurance payment required for full insurance conditional on the negative signal, $\underline{\pi}$. Since $\underline{\pi} < \pi$, outsiders need fewer resources than safe sellers would require. Consequently, it suffices for outsiders to possess at least the same level of resources as safe sellers.

Proposition 9 also establishes a condition ensuring that the threshold on γ is economically meaningful. This requires a sufficiently severe fragility shock (small ϕ). Under this condition, safe sellers can provide insurance against buyers' endowment risk and absorb the CCP's default losses even when only a negligible fraction of sellers defaults. If, instead, this condition is not satisfied, two degenerate cases emerge: replacement becomes either universally infeasible across all $\gamma \in (0, 1)$ due to insufficient resources among safe sellers, or universally feasible due to well-capitalized fragile sellers (high ϕ).

The optimal margin α^* equals the fragility shock ϕ (in the limit). This highlights the role of margins as *canaries in the coal mine*: the smaller the difference between fragile and safe sellers, the larger the optimal margin required to distinguish between seller types.

We now investigate the conditions under which the optimal contract with replacement (characterized by Propositions 8 and 9) provides greater utility than the contract without early replacement of fragile counterparties. Although replacement improves risk sharing, it is not unambiguously efficient. First, the risk sharing provided by outsiders is constrained by the Hirshleifer effect: outsiders can provide insurance only after information about the endowment risk has been revealed. Second, replacement of fragile counterparties is costly: it requires safe sellers to post margins and triggers default and replacement costs. Consequently, a contract *without* early default and replacement may provide greater utility to buyers.

To assess the efficiency of replacement, we assume that the full-insurance contract with replacement is feasible on the interval $(0, \overline{\gamma}^R]$:

Assumption 3. *Assume that $\phi < \phi^R$ and $\underline{\pi} \leq R_O$.*

We are now in a position to compare the optimal contracts with and without replacement.

Proposition 10 (Efficiency of Counterparty Replacement)**.**

(1) If $\phi < \widehat{\phi}$ and $C < \widehat{C}$, the contract with counterparty replacement enables full insurance for a larger fraction of fragile sellers than the contract without replacement ($\overline{\gamma}^{NR} < \overline{\gamma}^R$), where $\widehat{C}, \widehat{\phi} > 0$.

(2) If $\overline{\gamma}^{NR} < \gamma \leq \overline{\gamma}^R$ and $m^R - m^{NR} \leq g^{NR}$, counterparty replacement is efficient, where m^R and m^{NR} denote the markups with and without replacement, respectively, and $g^{NR} > 0$ denotes the consumption risk premium under the optimal contract without replacement.

The first part of Proposition 10 shows that replacement increases the CCP's risk-sharing capacity when deadweight replacement costs C are sufficiently small. Specifically, we compare the maximum proportion of fragile sellers γ for which full insurance can be achieved without replacement ($\overline{\gamma}^{NR}$) with the corresponding threshold with replacement ($\overline{\gamma}^R$). When replacement costs C are small, replacement allows the CCP to implement full risk sharing for a larger proportion of fragile counterparties: $\overline{\gamma}^{NR} < \overline{\gamma}^R$. This result highlights the key benefit of replacement: transferring contracts from fragile sellers to outsiders generates additional risk sharing and thereby reduces the loss-sharing burden on safe sellers.

The second part of Proposition 10 shows that when the mass of fragile sellers γ lies in the interval $(\overline{\gamma}^{NR}, \overline{\gamma}^R]$ and the markup with replacement is sufficiently small, replacement of fragile counterparties is efficient. In this case, the optimal contract with replacement implements greater risk sharing (full insurance) than the contract without replacement (partial insurance). The markup with replacement m^R is composed of deadweight replacement costs C, default costs $(1-\rho)\phi R(\underline{s})$, and opportunity costs of margins $(1-k)\alpha R(\underline{s})$ (see Proposition 8). In contrast, without replacement, the markup m^{NR} is either equal to zero because no seller defaults or it is composed of default costs and opportunity costs of margins. Therefore, the markup without replacement may be lower especially when no sellers default or margin requirements are low. Nonetheless, risk-averse buyers are willing to pay a strictly positive premium $g^{NR} > 0$ to eliminate the consumption risk that is present in the absence of replacement. Thus, replacement is efficient when the markup differential $m^R - m^{NR}$ is smaller than g^{NR}, as in this case the risk-sharing benefits outweigh the additional costs. This condition is trivially satisfied when the markup differential is negative.[13]

In contrast, when there are few fragile sellers ($\gamma \leq \overline{\gamma}^{NR}$), replacement is inefficient. In this case, the optimal contract without replacement already provides full insurance, making replacement unnecessary since it would only add costs without improving risk sharing. Therefore, it is efficient to forgo replacement and rely on ex-post loss sharing at $t = 2$.

Figure 1 illustrates the resulting pecking order of full-insurance contracts. On the interval $[0, \overline{\gamma}^{NR}]$, it is optimal to rely on ex-post loss sharing among surviving clearing members to implement full insurance. On the interval $(\overline{\gamma}^{NR}, \overline{\gamma}^{R}]$, full insurance with replacement is efficient provided that the risk-sharing benefit exceeds the required contract markup. When $\gamma > \overline{\gamma}^R$, safe sellers lack sufficient resources to implement full insurance with replacement and absorb the resulting default losses, so risk sharing must be reduced to prevent safe sellers from defaulting at $t = 2$.

We illustrate this pecking order of contracts in the following example, focusing on the case of very fragile sellers.

Example 1 (Very fragile sellers)**.** *We consider the case where $\phi = 0$ (i.e., fragile sellers lose all their assets due to the fragility shock) and $C = 0$ (i.e., there are no deadweight costs of replacement). In this case,*

$$\overline{\gamma}^R = \frac{R(\underline{s}) - \pi}{R(\underline{s}) - \underline{\pi}} \tag{34}$$

$$\textit{and } \overline{\gamma}^{NR} = \frac{R(\underline{s}) - \pi}{R(\underline{s})}. \tag{35}$$

[13] If deadweight replacement costs C are small and the CCP's recovery rate ρ is high, the markup with replacement may be smaller than without replacement. This occurs because only safe sellers post margin, reducing the opportunity costs of margin.

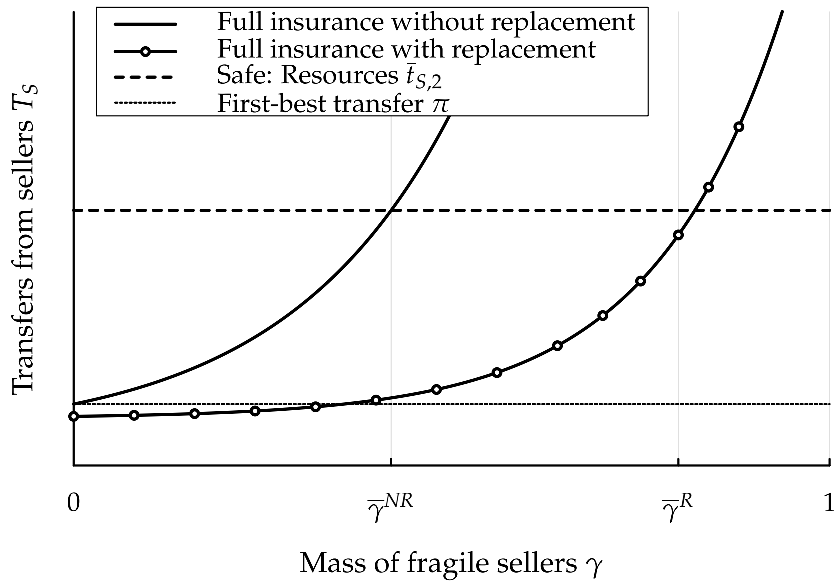

Figure 1: Full Insurance Pecking Order.
The figure plots the transfers $T_S(\underline{s}, \underline{e})$ made by sellers to the CCP after a negative signal in state $(\underline{s}, \underline{e})$ on contracts that provide full insurance (y-axis) against the probability γ of sellers to become fragile at $t = 1$ (x-axis). For ease of illustration we focus on the case that Condition (17) is globally satisfied, such that the optimal margin of a full-insurance contract without replacement is $\alpha^{NR} = 0$. The dashed line represents the maximum transfer $\bar{t}_{S,2}$ that safe sellers are able to make. The solid line represents transfers without replacement. It exceeds $\bar{t}_{S,2}$ for $\gamma > \overline{\gamma}^{NR}$, in which case the contract is not feasible. The line with diamonds represents transfers with replacement. It exceeds $\bar{t}_{S,2}$ for $\gamma > \overline{\gamma}^{R}$, in which case the contract is not feasible. In the interval $(0, \overline{\gamma}^{NR}]$ full insurance without margins is feasible without replacement and, thus, replacement is not efficient. In the interval $(\overline{\gamma}^{NR}, \overline{\gamma}^{R}]$ full insurance is feasible only with replacement.

If $\overline{\gamma}^{NR} < \gamma \leq \overline{\gamma}^{R}$, the optimal contract with replacement and defaults at $t = 1$ is resource-compatible and dominates the optimal contract without replacement.

Example 1 illustrates that replacement at $t = 1$ is efficient when there are sufficiently many fragile sellers (high γ) and both fragile sellers' default costs ϕ and deadweight replacement costs C are sufficiently small. In this case, replacement enables the CCP to provide full insurance to protection buyers with a modest margin requirement and, therefore, a small markup. The optimal contract without replacement provides only partial insurance to buyers and is dominated by the contract with replacement.

In the example, the maximum default risk for which full insurance is feasible, $\overline{\gamma}^{R}$, is increasing in $\underline{\pi}$. Thus, a less informative signal at $t = 1$, which increases $\underline{\pi}$, increases the benefits of replacement by preserving more scope for risk sharing. More generally, signal informativeness affects the value of replacement through two channels. First, a more informative signal reduces the risk that can be shared with outsiders, which we refer to as the Hirshleifer (1971) effect. Second, it changes the ex-ante probability of observing a negative signal and, therefore, the markup required by sellers. We refer to this second channel as the ex-ante pricing effect. In addition to the case considered in Example 1, the following proposition provides a sufficient condition under which the Hirshleifer effect dominates.

Proposition 11 (Signal informativeness)**.** *If $\overline{\gamma}^{NR} < \overline{\gamma}^{R}$ and $\pi \geq 1/2$, the additional risk-sharing capacity achieved by replacement is decreasing in signal informativeness λ.*

Signal informativeness determines the strength of the Hirshleifer effect. The more information is revealed about $\tilde{e}$, the more sensitive the value of the derivative contract becomes to the signal. In particular, a more informative signal reduces $\underline{\pi}$ and therefore increases $\pi - \underline{\pi}$. Outsiders then provide less insurance after a negative signal, increasing the CCP's default loss and the contribution required from safe sellers to loss sharing (see Proposition 8). As a result, the maximum proportion of fragile sellers for which full insurance with replacement is feasible, $\overline{\gamma}^{R}$, decreases.

The ex-ante pricing effect works through the ex-ante probability of a negative signal. If $\pi < 1/2$, a more informative signal increases $\mathbb{P}(\underline{s})$ because the low endowment realization is more likely ex ante. Defaults and replacement then occur more frequently, increasing the markup that sellers require to enter the contract. A larger markup reduces the transfers made by safe sellers and thereby relaxes their resource constraint, partly offsetting the Hirshleifer effect. If instead $\pi \geq 1/2$, a more informative signal weakly reduces $\mathbb{P}(\underline{s})$, such that the ex-ante pricing effect reinforces the Hirshleifer effect. Moreover, if $\phi = C = 0$, replacement entails neither default nor replacement costs and the contract is actuarially fair. In this case, the ex-ante pricing effect disappears.

A higher signal informativeness λ can be interpreted as a situation in which more information about the underlying payoff has been revealed by the time fragility becomes apparent. As maturity approaches, public signals about final cash flows become more informative. From this perspective, Proposition 11 suggests that counterparty replacement is most useful as an early intervention tool, before much information about final cash flows has been revealed. Similarly, replacement is more valuable when the fragility shock affecting counterparties is only imperfectly correlated with the value of their derivative positions (see Section 7.2 for a more detailed discussion).

5.2 Partial Insurance

When there are too many fragile sellers ($\gamma > \overline{\gamma}^{R}$), counterparty replacement fails to restore full insurance. Instead, the optimal contracts with and without replacement only provide partial insurance. In the following, we first characterize the optimal contract with replacement for this case and then compare it to the one without replacement.[14]

Proposition 12 (Partial insurance with replacement)**.** *Suppose that $\gamma > \overline{\gamma}^R$ and that the resource constraint of safe sellers is slack in state $(\underline{s}, \overline{e})$ in equilibrium. The optimal contract with default and replacement of fragile sellers at $t = 1$ provides partial insurance to buyers, with*

$$u(\underline{s}, \underline{e}) < u(\underline{s}, \overline{e}) < u(\overline{s}, \underline{e}) = u(\overline{s}, \overline{e}). \tag{36}$$

In particular, optimal risk sharing after a negative signal satisfies

$$u'(\underline{s}, \overline{e}) = \frac{\gamma(1 - \underline{\pi})}{1 - \gamma\underline{\pi}} u'(\underline{s}, \underline{e}) + \frac{1 - \gamma}{1 - \gamma\underline{\pi}} u'(\overline{s}, \overline{e}). \tag{37}$$

The contract is not actuarially fair but entails a markup paid by buyers of

$$m = \mathbb{P}(\underline{s}) \Big(\gamma C + \gamma(1 - \rho)\phi R(\underline{s}) + (1 - \gamma)(1 - k)\phi R(\underline{s}) \Big). \tag{38}$$

Safe sellers' resource constraint binds in state $(\underline{s}, \underline{e})$:

$$T_S(\underline{s}, \underline{e}) = \big(1 - (1 - k)\phi\big) R(\underline{s}). \tag{39}$$

The optimal margin remains the smallest margin that induces fragile sellers to default at $t = 1$,

$$\alpha = \alpha^* = \phi. \tag{40}$$

When safe sellers' resources become scarce, buyers optimally absorb some of the CCP's default loss. In contrast to the full-insurance contract, consumption is equalized neither across endowment realizations following a negative signal nor across signal realizations. Specifically, even conditional on $\underline{s}$, buyers are no longer fully insured against endowment risk, so that $u(\underline{s}, \underline{e}) < u(\underline{s}, \overline{e})$. The optimal margin remains at the screening threshold because any additional margin would impose additional opportunity costs on surviving safe sellers without increasing the resources recovered from fragile sellers, who default before posting margin.

In contrast to the case without replacement, the binding resource constraint of safe sellers reduces buyer consumption not only in the worst state $(\underline{s}, \underline{e})$ but also in $(\underline{s}, \overline{e})$. The

[14] We focus on the case in which safe sellers' resource constraint binds only in $(\underline{s}, \underline{e})$ but not in $(\underline{s}, \overline{e})$. At $\gamma = \overline{\gamma}^R$, the safe-seller resource constraint binds in $(\underline{s}, \underline{e})$ but is strictly slack in $(\underline{s}, \overline{e})$, since full insurance implies $T_S(\underline{s}, \underline{e}) - T_S(\underline{s}, \overline{e}) = 1$. Hence, by continuity, the constraint remains slack for γ in an interval above $\overline{\gamma}^R$.

reason is that, with replacement, safe sellers' resources provide insurance along two dimensions: they insure buyers against the endowment realization, which constrains their resources particularly in $(\underline{s}, \underline{e})$, and they absorb the CCP's default loss following a negative signal, thereby insuring buyers against signal risk. When safe sellers become constrained, it is optimal to reduce insurance along both dimensions.

Reducing consumption in $(\underline{s}, \overline{e})$ has an additional benefit under replacement. A lower seller transfer in that state reduces the market value of the contract at $t = 1$, $\mathbb{E}[T_S \mid \underline{s}]$, and hence the price paid to outsiders who take over the defaulted positions. The CCP's default loss following a negative signal shrinks accordingly, relaxing the resource constraint in the worst state $(\underline{s}, \underline{e})$. Buyer consumption in $(\underline{s}, \overline{e})$ therefore optimally falls below consumption after a positive signal.

Without replacement, buyer consumption is equalized across all states except $(\underline{s}, \underline{e})$, whereas the optimal contract with replacement has three distinct consumption levels. This difference makes a general ranking of the two contracts difficult. We therefore derive a sufficient condition for replacement to be efficient by considering a restricted replacement benchmark that imposes the same two-level consumption structure as the optimal contract without replacement. Specifically, the benchmark maximizes buyer utility subject to the constraints of the replacement problem and the additional restriction that consumption is equalized across all states except $(\underline{s}, \underline{e})$:

$$1 - T_B(\underline{s}, \overline{e}) = 1 - T_B(\overline{s}, \overline{e}) = -T_B(\overline{s}, \underline{e}). \tag{41}$$

The restriction can only reduce buyer utility. Hence if the restricted benchmark dominates the optimal contract without replacement, so does the unrestricted optimal contract with replacement, and replacement is efficient.

Proposition 13 (Efficiency of counterparty replacement with constrained sellers)**.** *Suppose that $\gamma > \overline{\gamma}^R$ and that the optimal contract without replacement induces defaults by fragile sellers at $t = 2$. Denote the gap between buyer consumption in state $(\underline{s}, \underline{e})$ and the remaining states under the restricted replacement benchmark by $\Delta^{R,res}$ and that under the optimal contract without replacement by Δ^{NR}.*

If

$$m^R \leq m^{NR}, \tag{42}$$
$$\Delta^{R,res} \leq \Delta^{NR}, \tag{43}$$

with at least one inequality strict, then the optimal contract with counterparty replacement domi-

nates the optimal contract without replacement.

If $\phi = 0$ and $C = 0$, the optimal contract with replacement strictly dominates the optimal contract without replacement.

Proposition 13 provides a sufficient condition for replacement to be efficient. The restricted replacement benchmark removes the adjustment in $(\underline{s}, \bar{e})$ that is part of the optimal replacement contract and therefore provides weakly lower utility than the optimal contract. If replacement dominates under this restriction, it must also dominate without it.

The polar case of $\phi = 0$ and $C = 0$ illustrates the benefit of replacement particularly clearly. With $\phi = 0$, fragile sellers have no resources left after the fragility shock, and with $C = 0$ transferring their contracts to outsiders entails no deadweight cost. The canary margin is then arbitrarily small, so that the contract with replacement does not entail a markup ($m^R = 0$). In contrast, ring-fencing margins may lead to a positive markup without replacement ($m^{NR} \geq 0$). Because outsiders take over the positions of fragile sellers and provide additional insurance conditional on the negative signal, replacement improves risk sharing ($\Delta^{R,res} < \Delta^{NR}$). Replacement therefore leads to an overall increase in buyer utility. By continuity, it remains efficient if ϕ and C are sufficiently small.

6 Extensions

6.1 Replacement by Incumbent Sellers

The baseline model assumes that outsiders enter at $t = 1$ to replace fragile sellers. In Appendix B, we show that our main result (i.e., replacement is efficient when fragility is sufficiently severe and widespread) does not rely on the availability of outside sellers at $t = 1$. Specifically, we consider an alternative model in which all sellers are present and identical at $t = 0$ but, following a negative signal at $t = 1$, privately learn whether they are fragile, mediocre, or safe. These three types differ only in their balance sheet capacity, with fragile sellers having the fewest resources and safe sellers the most.

When only few sellers are fragile, mediocre and safe sellers can absorb fragile sellers' default losses through loss sharing and maintain full insurance, as in Proposition 5. As the mass of fragile sellers increases, the loss sharing contribution required from surviving sellers increases as well. Once this contribution exceeds mediocre sellers' resources, they default together with the fragile sellers, concentrating the remaining losses on safe sellers. If safe sellers also lack sufficient resources to absorb these losses, full insurance is no longer feasible and the CCP would fail, as in Proposition 6.

Replacement allows the CCP to redirect the positions of fragile sellers to the sellers with the largest balance sheets. For this purpose, the CCP first uses the canary margin to induce fragile sellers to default. As in the baseline model, this margin is set just high enough to exhaust fragile sellers' resources. This first margin identifies fragile sellers, but does not distinguish mediocre from safe sellers. The CCP therefore imposes an additional margin requirement on sellers that take over the defaulted positions. This second margin is set just high enough that mediocre sellers cannot take on the additional positions, while safe sellers can. Hence both margins serve as screening devices: the first identifies the sellers that should be replaced, while the second selects the sellers with sufficient balance sheet capacity to replace them.

We show that this mechanism can expand the scope for risk sharing. To characterize when replacement is efficient, we consider a benchmark in which fragile sellers are completely impaired ($\phi = 0$). In this case, their default destroys no resources, so allowing fragile sellers to default at maturity strictly dominates preventing their default. The optimal contract without replacement therefore takes one of two forms. Either only fragile sellers default while mediocre and safe sellers remain solvent, or both fragile and mediocre sellers default.

Replacement can be efficient in both cases. When only fragile sellers default without replacement, replacement eliminates the remaining consumption risk by reallocating their positions to safe sellers. This is efficient when the value of this additional insurance exceeds the opportunity cost of the two screening margins. The second margin allows mediocre sellers to retain their original positions while directing fragile sellers' defaulted positions to safe sellers.

When both fragile and mediocre sellers default without replacement, replacement has an additional benefit. By keeping mediocre sellers solvent, it avoids the deadweight losses associated with their default and preserves their resources for the positions they already hold. We provide conditions under which this benefit alone is sufficient to outweigh the opportunity cost of the screening margins, in which case replacement strictly dominates the optimal contract without replacement.

Overall, the extension shows that the risk sharing benefits of replacement do not require a separate pool of outside counterparties. They also arise within the original clearing-member pool when there is sufficient heterogeneity in balance sheet capacity.

6.2 Bilateral Trading

In the baseline model, contracts are centrally cleared and default losses can be shared among surviving sellers. Appendix C considers an alternative model in which buyers and sellers trade bilaterally. Each buyer is matched with one seller at $t = 0$ and remains exposed to that seller unless it is replaced at $t = 1$. If a fragile seller defaults at maturity $t = 2$, her buyer bears the default loss alone. A canary margin can induce fragile sellers to default at $t = 1$, allowing an outsider to take over their positions. Since replacement takes place after the signal $\tilde{s}$ has been observed, the outsider provides insurance against the remaining endowment risk conditional on the signal. Buyers therefore remain exposed to signal risk, which under central clearing is absorbed through loss sharing.

We show that replacement can also be efficient under bilateral trading. Specifically, we consider the benchmark with no replacement costs, $C = 0$, and completely impaired fragile sellers, $\phi = 0$. In this case, default at maturity entails no deadweight loss and the canary margin has no opportunity cost. If outsiders have sufficient resources, replacement strictly dominates bilateral contracts that either prevent default or allow it at maturity. Replacement preserves expected buyer consumption while reducing the consumption risk generated by the exposure to fragile sellers.

Replacement is feasible for any mass of fragile sellers $\gamma > 0$ provided that $R(\underline{s}) \geq \overline{\pi}$ and $R_O \geq \underline{\pi}$. In contrast to the case with central clearing, the benefit of replacement under bilateral trading therefore does not require fragility to be widespread. The reason is that without loss sharing, even a single fragile counterparty exposes its buyer directly to default risk and, thus, replacement is beneficial whenever an alternative counterparty is available.

This comparison isolates the contribution of central clearing: Bilateral replacement works for any γ, but delivers full insurance only conditional on the signal, leaving buyers exposed to signal risk. Mutualization removes that exposure and delivers unconditional full insurance, at the cost of an aggregate loss-sharing burden that makes full insurance infeasible once fragility is sufficiently widespread, $\gamma > \overline{\gamma}^R$.

6.3 Fragility after a Positive Signal

In our baseline analysis, we assume that resources are abundant after a positive signal, so that sellers' constraints are slack and they do not default. If instead fragile sellers default after $\tilde{s} = \overline{s}$, the logic of loss sharing is similar to that after a negative signal, and canary margins may again be optimal at $t = 1$. There is, however, one key difference. After a positive signal, the contract may be an asset rather than a liability for the seller at $t = 1$, $\mathbb{E}[T_S \mid \bar{s}] < 0$. Outsiders are then willing to pay to acquire it, so fragile sellers can transfer their positions and avoid defaulting on the CCP's margin call. Canary margins remain useful, but they now induce fragile sellers to sell their positions directly to outsiders rather than to default and be replaced.

7 Discussion and Policy Implications

CCPs manage counterparty default risk using three tools: margins, replacement, and loss sharing. In the following, we discuss the implications of our results for each of these.

7.1 Margin Setting by CCPs

Our model highlights a novel role of margin requirements as canaries in the coal mine. It is useful to briefly reflect on the key differences and similarities between the two economic roles of margins: as ring-fenced collateral versus as an early intervention mechanism to replace counterparties.[15]

Margins as ring-fencing. Conventional wisdom views margins as a tool to increase the resources available to CCPs in the event that some clearing members default. For example, Hull (2012, p. 30) describes the role of margins as follows: "The whole purpose of the margining system is to ensure that funds are available to pay traders when they make a profit." According to this view, margins enable the CCP to seamlessly seize collateral from defaulted counterparties, thereby increasing the resources available to pay those who are owed payments.

Note, however, that seizing collateral improves outcomes only in cases where uncollateralized loss-sharing arrangements are insufficient to make counterparties whole. As long as safe clearing members can provide sufficient resources to the CCP, loss sharing among clearing members ensures that contractual payments are made, even when fragile clearing members default. This is the case as long as there are not too many fragile clearing members ($\gamma \leq \overline{\gamma}^{NR}$ in Figure 1). In this case, margins are not needed to reduce the burden on surviving sellers. However, it may still be optimal to use margins if the CCP has difficulty seizing defaulters' assets otherwise (low ρ), reducing the deadweight costs resulting from defaults.

[15] The literature has discussed a third role of margins: reducing incentives to default. As shown by Biais et al. (2016), this "incentive role" of margins emerges when clearing members are subject to a moral hazard problem in managing their default risk. Because of the absence of moral hazard in our framework, this incentive role does not arise in our model.

In contrast, when there are sufficiently many fragile clearing members ($\gamma > \overline{\gamma}^{NR}$ in Figure 1), default losses exceed the surviving clearing members' resources and it becomes beneficial to also use margins to reduce the burden on surviving sellers. By ring-fencing some of the defaulters' assets, margins increase the resources the CCP can seize from defaulters.

Margins as canaries. Alternatively, the CCP can increase resources by replacing fragile clearing members with less fragile clearing members before contractual payments are due. Clearing membership requirements partly achieve this goal by ensuring that clearing members exhibit low *observable* default risk, such as a high credit rating. However, changes in credit quality are usually reflected in ratings only with a significant time lag and, at high frequency, remain *unobservable* to the CCP.

Employing margins as canaries in the coal mine overcomes this obstacle. A sufficiently large margin call identifies fragile clearing members by forcing them to default. Because these defaults occur before contract expiry, the CCP can replace fragile clearing members with new counterparties, thereby increasing the CCP's resources in adverse states. CCPs acknowledge this value of using margins as canaries: "While sufficient time should be granted to avoid unnecessary liquidity strains for market participants to meet the ITD [intraday] margin calls, allowing excessive time for them to arrange funding may impede the timely identification of a default. This is particularly true when a CM is experiencing solvency issues and struggling to meet collateral requirements, potentially leading to a default."[16]

As with the ring-fencing role of margins, using a margin call as a canary is not required when loss sharing among safe clearing members generates sufficient resources for the CCP to honor contractual payments ($\gamma \leq \overline{\gamma}^{NR}$ in Figure 1). In contrast, when the CCP's resources are constrained, it can be efficient to use canary margins.

To induce defaults, margin requirements must be sufficiently large. This implies that canary margins generally exceed ring-fencing margins, resulting in higher opportunity costs. In addition, using margins as canaries leads to deadweight costs resulting from the defaults of fragile clearing members. Therefore, the total costs of canary margins can exceed those of margins used to ring-fence collateral. However, these additional costs can be outweighed by the benefits of replacement, namely that outside sellers provide additional risk-sharing capacity. This benefit is particularly large when, at the time of the fragility shock, relatively little information about the underlying risk has been revealed

[16]CCP Global, Re "CPMI-IOSCO Report on Streamlining variation margin in centrally cleared markets-examples of effective practices", 13 May, 2024

(low λ), as outsiders can only insure risks that have not yet materialized.

Distinguishing ring-fencing and canary margins. In general, ring-fencing margins and canary margins differ in two key dimensions: canary margins are weakly larger and are explicitly designed to trigger defaults. While clearing member defaults are infrequent events, recent defaults have indeed been associated with margin calls rather than payments at contract maturity.[17]

The relative effectiveness of these margin types depends on the degree of counterparty fragility. When clearing members retain substantial balance sheet capacity, ring-fencing margins efficiently secure collateral without triggering defaults. However, when balance sheets become severely impaired—precisely when ring-fencing becomes ineffective due to insufficient collateralizable resources—canary margins become the optimal tool. Even relatively small margin requirements will induce fragile counterparties to default, enabling their replacement with better-capitalized outsiders. Thus, the CCP's optimal margin strategy shifts from ring-fencing to screening as counterparty fragility increases.

Initial and variation margin. In practice, margins are set according to contract and clearing member characteristics, with two types of margin requirements. Initial margin is based on the risk (e.g., volatility or Value-at-Risk) of the position as well as the default risk of the counterparty. Initial margin is posted at initial contracting and can be updated throughout the life of the contract. Variation margin is based on the market value of the contract (often zero at inception) and is usually adjusted on a daily basis.

In the context of our model, variation margin at $t = 1$ equals the expected liability arising from the original (bilateral) derivative contract, which equals the expected total net transfers less the loss-sharing contributions:

$$VM_1 = \mathbb{E}[T_S \mid \underline{s}] - L^{CCP} = \pi - \underline{\pi} - m. \tag{44}$$

Under the contract with replacement, the canary margin equals $\phi R(\underline{s})$.[18] Therefore, when fragile counterparties are sufficiently impaired ($\phi R(\underline{s}) \leq \pi - \underline{\pi} - m$), variation margin alone is sufficient to trigger their default at $t = 1$. Otherwise, an additional initial margin is needed to induce default and replacement of fragile counterparties. Combining both cases, the total initial margin demanded at $t = 1$ to trigger replacement of fragile

[17]See Section 7.2 for an example.

[18]More precisely, the margin is arbitrarily close to ϕ, see Equation (31).

counterparties is given by

$$IM_1 = \max\{\phi R(\underline{s}) - (\pi - \underline{\pi} - m), 0\}. \tag{45}$$

This initial margin can be interpreted as a credit risk add-on that is required only after some sellers have suffered a fragility shock. However, unlike the conventional ring-fencing role of margins, this margin in the contract described in Section 5 functions as a screening device to induce default and enable replacement of fragile sellers with healthier counterparties.

Procyclicality. Policymakers are concerned about margin procyclicality, namely that larger margins during turbulent times may trigger defaults when other market participants are also more fragile.[19] Our model suggests a more nuanced perspective on this concern. First, higher margins can be a means for CCPs to deal with fragility of clearing members by enabling replacement. However, an important precondition is that alternative counterparties ("outsiders") are sufficiently unconstrained ($R_O \geq \underline{\pi}$). Thus, the effects of procyclicality depend on the heterogeneity of fragility across market participants. Procyclical margins can be detrimental if all market participants are fragile, but beneficial when some market participants remain safe.

Second, the effectiveness of counterparty replacement depends on how correlated a fragility shock to clearing members is with the risks of the contracts cleared by the CCP. Replacement counterparties are less willing to insure endowment risk when more information about this risk has been revealed (i.e., λ is high). This is likely a particular concern during crisis times (e.g., credit default swaps during the 2008 global financial crisis). Accordingly, procyclical margins are more likely to be beneficial as a tool for early intervention rather than in the midst of a crisis.

Overall, our analysis therefore suggests that optimal margin-setting policies should account for both the heterogeneity of market participants and the information environment when evaluating procyclical adjustments.

7.2 Counterparty Replacement

A key result of our analysis is that default and subsequent replacement of counterparties can improve risk sharing and prevent cascading defaults of clearing members. To

[19]For example, see the final report on the "Review of the RTS with respect to the procyclicality of CCP margin" of the European Securities and Markets Authority (2023).

demonstrate the practical relevance of this insight, we now briefly discuss how counterparty replacement is implemented in practice.

CCP rules. CCP rules prescribe the conditions that trigger the default of a clearing member and the procedures to be followed in the event of defaults. In particular, if a clearing member is (or appears to be) unable to meet its obligations to the CCP (such as posting margin), the CCP may terminate the clearing membership (i.e., put the clearing member into default). In the event of such a default, the CCP can take several steps to neutralize the defaulter's positions. One of these is to enter offsetting positions or to transfer the defaulter's positions to an alternative counterparty, which replaces the defaulted counterparty and effectively takes the role of the "outsiders" in our model.

This replacement mechanism occurs regularly in practice. For example, in 2018, the Swedish clearinghouse Nasdaq Clearing AB declared the power trader Einar Aas in default after he failed to post required margins. The CCP then conducted an auction to transfer the trader's positions to a new counterparty–essentially implementing the counterparty replacement mechanism that our model shows can improve overall risk sharing.[20]

The timing of counterparty replacement. Our model shows that the benefits of replacement crucially depend on the timing of defaults and replacement. In our framework, this corresponds to interpreting the informativeness of the signal λ as capturing the proximity to contract maturity. The earlier fragile counterparties default, the larger the remaining scope for risk sharing. In the extreme case where fragile counterparties default at contract maturity (equivalent to $\lambda = 1$), replacement is not useful because outsiders would be willing to enter the contract only if they were paid exactly the cash flow that is due to be paid by them. Instead, when replacement occurs before contract expiry, the final payment remains risky and thus outsiders are willing to provide insurance against this risk.

Moreover, replacement is more beneficial when relatively unconstrained outsiders are available to take on the positions of defaulted clearing members ($R_O \geq \underline{\pi}$). Only then is it possible for outsiders to absorb the positions of defaulters without increasing their own

[20]See https://view.news.eu.nasdaq.com/view?id=b342623f1e259ec1892c5e1763ad7de15&lang=en, https://www.finanstilsynet.no/en/news-archive/news/2018/nasdaq-clearing-ab-default-of-clearing-member/, and https://www.bis.org/publ/qtrpdf/r_qt1812x.htm for the Einar Aas default. Similarly, Klein and Co. Futures, Inc. failed to meet a margin call to the New York Clearing Corporation (NYCC), with liquidation expenses leading to a default loss of approximately $6 million (as described in ICE's 10-Q filing from July 2007). See McPartland and Lewis (2017) for examples of other clearing member defaults.

default risk. Thus, replacement is most useful when defaults by fragile counterparties are idiosyncratic and uncorrelated with the constraints of other market participants. If outsiders are constrained exactly when fragile sellers default, replacement is less likely to improve outcomes. This suggests that the effectiveness of canary margins depends critically on market-wide stress conditions.

7.3 Loss sharing

Finally, our analysis provides some perspectives on default losses and how they are shared within a CCP.

Default losses and the default waterfall. If the market value of the contracts to be transferred is negative, the counterparties that take on these contracts need to be compensated for the (negative) value of these positions. Therefore, when the CCP transfers a defaulter's positions to new counterparties, the CCP usually incurs a default loss. In our model, this default loss is increasing in $\pi - \underline{\pi}$, which is larger the more informative the signal at $t = 1$. This loss reflects the Hirshleifer effect: more informative signals reduce the insurance that outsiders can provide, increasing the compensation they require to assume defaulted positions.

CCPs absorb default losses according to a predetermined default waterfall. First, the defaulter's resources, such as previously posted collateral, are used to the extent that the CCP can access them. Second, the CCP typically provides a layer of its own equity to absorb additional losses. Our model abstracts away from the CCP's equity contribution, which is typically small in practice.[21] All remaining default losses are then absorbed by the surviving clearing members through loss-sharing arrangements. This loss sharing is the focus of our theoretical analysis.

Loss sharing rules. In our model, default losses should optimally be allocated to risk-neutral protection sellers, who are better positioned to bear risk than risk-averse protection buyers and thus willing to insure them against the risk of CCP default losses. This allocation principle provides a benchmark for evaluating actual CCP loss-sharing arrangements.

[21] Kuong and Maurin (2023) show that when the interests of CCPs and their members are not perfectly aligned, equity provides incentives for CCPs to monitor counterparty risk. Our model abstracts from such agency conflicts by assuming the CCP maximizes member welfare, allowing us to focus on the screening role of margins.

In practice, the counterparties that are best able to bear this risk are typically dealers with large balance sheets. However, current loss-sharing mechanisms often imply the opposite. For example, some CCPs assign losses proportionally to net exposures. Dealers acting as intermediaries often have close-to-zero net exposure, so that they contribute little to loss sharing relative to the size of their portfolios.[22]

8 Conclusion

This paper shows that counterparty replacement is central to how CCPs optimally manage counterparty risk when their loss-sharing capacity is limited. Without replacement, defaults at contract maturity can overwhelm surviving clearing members and trigger cascading failures. Replacing fragile counterparties before maturity can prevent this outcome by transferring their positions to counterparties with more resources while risk sharing remains possible. Margin calls enable this replacement by serving as canaries in the coal mine: rather than deterring default, they can induce fragile counterparties to default early and reveal which positions should be transferred. Our model therefore highlights the complementary roles of margins, counterparty replacement, and loss sharing in CCP risk management.

Our findings have broad implications for the post-crisis financial architecture that places CCPs at the center of derivatives markets. The effectiveness of central clearing depends not only on the CCP's ability to pool default losses among surviving members, but also on its capacity to replace weak counterparties before their failure exhausts this loss-sharing capacity. This perspective is particularly relevant for margin procyclicality: higher margins may enhance rather than impair financial stability when they facilitate the replacement of weaker counterparties with stronger ones. It also suggests that regulatory frameworks should consider the availability of counterparties that can absorb defaulted positions, in addition to margins and default resources.

[22] Kubitza et al. (2024) highlight a related point driven by a different mechanism. They show that allocating default losses to end-investors rather than dealers reduces incentives to use central clearing. They also provide a model that highlights a CCP's incentives to assign losses to end-investors in order to attract large clearing volumes from dealers.

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A Proofs

A.1 First Best

Proof of Proposition 1. With abundant resources, the optimal contract maximizes expected utility of buyers (4) subject to the participation constraint of protection sellers (5) and the CCP's budget constraints (3). The first-order conditions are

$$-\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e}) + \eta(\tilde{s},\tilde{e}) = 0 \quad \forall \tilde{s},\tilde{e} \tag{A.1}$$

$$-\mathbb{P}(\tilde{s},\tilde{e})\xi + \eta(\tilde{s},\tilde{e}) = 0 \quad \forall \tilde{s},\tilde{e}, \tag{A.2}$$

where $\eta(\tilde{s},\tilde{e})$ is the Lagrange multiplier on the budget constraint (3) in state $(\tilde{s},\tilde{e})$ and ξ that on the participation constraint (5). Using the second set of first-order conditions in the first yields

$$-\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e}) + \mathbb{P}(\tilde{s},\tilde{e})\xi = 0 \tag{A.3}$$

$$\Leftrightarrow u'(\tilde{s},\tilde{e}) = \xi. \tag{A.4}$$

Therefore, the participation constraint is binding and buyer utility is constant across states, i.e., buyers are fully insured. This implies

$$\tilde{e} - T_B(\tilde{s},\tilde{e}) = \underbrace{\mathbb{E}[\tilde{e}]}_{=\pi} - \mathbb{E}[T_B] \quad \forall \tilde{s},\tilde{e}. \tag{A.5}$$

The budget constraints (3) jointly with the binding participation constraint (5) imply that

$$\mathbb{E}[T_B] = 0, \tag{A.6}$$

i.e., the contract is actuarially fair. Therefore, full insurance implies the following transfers from buyers

$$1 - T_B(\tilde{s}, \bar{e}) = \pi \Leftrightarrow T_B(\tilde{s}, \bar{e}) = 1 - \pi \tag{A.7}$$

$$\text{and } 0 - T_B(\tilde{s}, \underline{e}) = \pi \Leftrightarrow T_B(\tilde{s}, \underline{e}) = -\pi \tag{A.8}$$

and from sellers (using the budget constraints)

$$T_S(\tilde{s}, \bar{e}) = -T_B(\tilde{s}, \bar{e}) = -(1 - \pi) \tag{A.9}$$

$$\text{and } T_S(\tilde{s}, \underline{e}) = -T_B(\tilde{s}, \underline{e}) = \pi. \tag{A.10}$$

Neither using margins nor outside sellers can improve risk sharing. Because both are costly, it is optimal to not use them. □

A.2 Central Clearing Without Replacement

A.2.1 Without Defaults

Proof of Proposition 2. The optimal contract maximizes expected buyer utility (4) subject to the participation and resource constraints of protection sellers and the CCP budget constraints. Because sellers do not default conditional on a good signal, their participation constraint is given by

$$\mathbb{E}[\tilde{\phi}R(\tilde{s})] \leq \mathbb{P}(\bar{s})\mathbb{E}[(1-\alpha(\bar{s}))R(\bar{s}) + \alpha(\bar{s})kR(\bar{s}) - T_S(\bar{s}, \tilde{e}) \mid \bar{s}] + \mathbb{P}(\underline{s})\mathbb{E}[f_1(\underline{s}, \tilde{e}) \mid \underline{s}] \tag{A.11}$$

where $f_1(\underline{s}, \tilde{e})$ is a function of transfers and margins conditional on a bad signal, $T_S(\underline{s}, \tilde{e})$ and $\alpha(\underline{s})$. The CCP's budget constraints conditional on a good signal are

$$T_S(\bar{s}, \tilde{e}) + T_B(\bar{s}, \tilde{e}) = 0 \quad \forall \tilde{e}, \tag{A.12}$$

whereas the budget constraints conditional on a bad signal do neither depend on $T_S(\bar{s}, \tilde{e})$, $T_B(\bar{s}, \tilde{e})$, nor $\alpha(\bar{s})$. Similarly, resource constraints after a negative signal $\underline{s}$ do neither depend on $T_S(\bar{s}, \tilde{e})$, $T_B(\bar{s}, \tilde{e})$, nor $\alpha(\bar{s})$.

The partial derivative of the Lagrangian with respect to $\alpha(\bar{s})$ is $-\mathbb{P}(\bar{s})(1-k)\xi R$, where ξ is the multiplier on the participation constraint. Due to a binding participation constraint (shown below), it is $-\mathbb{P}(\bar{s})(1-k)\xi R < 0$, and, therefore, no margin is used after a good signal: $\alpha(\bar{s}) = 0$.

The first-order conditions with respect to $T_B(\bar{s}, \tilde{e})$ and $T_S(\bar{s}, \tilde{e})$, respectively, are

$$-\mathbb{P}(\bar{s}, \tilde{e})u'(\bar{s}, \tilde{e}) + \eta(\bar{s}, \tilde{e}) = 0 \tag{A.13}$$

$$\eta(\bar{s}, \tilde{e}) - \xi\mathbb{P}(\bar{s}, \tilde{e}) = 0, \tag{A.14}$$

where $\eta(\bar{s}, \tilde{e})$ is the Lagrange multiplier on the budget constraint in state $(\bar{s}, \tilde{e})$.

Re-arranging (A.13) and (A.14) gives

$$-\mathbb{P}(\bar{s}, \tilde{e})u'(\bar{s}, \tilde{e}) + \xi\mathbb{P}(\bar{s}, \tilde{e}) = 0 \tag{A.15}$$

$$\Leftrightarrow \xi = u'(\bar{s}, \tilde{e}). \tag{A.16}$$

Therefore, $u'(\bar{s}, \underline{e}) = u'(\bar{s}, \bar{e})$, which implies that

$$\tilde{e} - T_B(\bar{s}, \tilde{e}) = \mathbb{E}[\tilde{e} - T_B(\bar{s}, \tilde{e})] \quad \forall \tilde{e}. \tag{A.17}$$

□

Proof of Proposition 3. The optimal contract maximizes expected buyer utility (4) subject to the CCP's budget constraints (3), and the participation constraint (8) and resource constraints (9) and (10) of protection sellers. Therefore, the partial derivatives of the Lagrangian with respect to $T_B(\tilde{s}, \tilde{e})$, $T_S(\tilde{s}, \tilde{e})$, and α, respectively, are (assuming that the $t = 1$ resource constraints are not binding)

$$-\mathbb{P}(\tilde{s}, \tilde{e})u'(\tilde{s}, \tilde{e}) + \eta(\tilde{s}, \tilde{e}), \tag{A.18}$$

$$\eta(\tilde{s}, \tilde{e}) - \xi\mathbb{P}(\tilde{s}, \tilde{e}) - (\sigma_{S,2} + \sigma_{F,2})1_{(\tilde{s},\tilde{e})=(\underline{s},\underline{e})}, \tag{A.19}$$

$$-\xi\mathbb{P}(\underline{s})(1-k)R(\underline{s}) - (\sigma_{S,2} + \sigma_{F,2})(1-k)R(\underline{s}) - (\sigma_{S,1} + \sigma_{F,1}), \tag{A.20}$$

where $\eta(\tilde{s}, \tilde{e})$ is the Lagrange multiplier on the budget constraint in state $(\tilde{s}, \tilde{e})$, $\sigma_{j,t}$ that on the resource constraint for sellers of type j at time t, and ξ that on the participation constraint.

Risk sharing We focus on an equilibrium in which no resource constraint is binding, $\sigma_{j,t} = 0$. In this case, (A.20) implies that there is no interior solution for α but, instead, there is no margin call: $\alpha = 0$. For an interior solution, (A.18) and (A.19) are equal to zero. Combining these yields

$$-\mathbb{P}(\tilde{s}, \tilde{e})u'(\tilde{s}, \tilde{e}) + \xi\mathbb{P}(\tilde{s}, \tilde{e}) = 0 \tag{A.21}$$

$$\Leftrightarrow u'(\tilde{s}, \tilde{e}) = \xi. \tag{A.22}$$

Therefore, the participation constraint is satisfied with equality and buyer consumption is independent of the signal and endowment risk realization, $\tilde{e} - T_B = \mathbb{E}[\tilde{e} - T_B]$.

Transfers The budget constraints (3) imply that

$$\mathbb{E}[T_B] = -\mathbb{E}[T_S], \tag{A.23}$$

which we use in the participation constraint (8):

$$0 = -\mathbb{E}[T_S] = \mathbb{E}[T_B]. \tag{A.24}$$

Using this above and that $\mathbb{E}[\tilde{e}] = \pi \cdot 1 + (1 - \pi) \cdot 0 = \pi$ gives the transfers of the first-best contract in Proposition 1.

No defaults at $t = 1$ Because $\phi_F < \phi_S = 1$, if the resource constraint (9) is satisfied for fragile sellers, it also holds for safe sellers. Because $\alpha = 0$ and $\phi_F \geq 0$, it is $\alpha \leq \phi_F$ and, thus, there are no defaults at $t = 1$.

No defaults at $t = 2$ Because $\phi_F < \phi_S = 1$, available resources at $t = 2$ are smaller for fragile sellers: $\bar{t}_{F,2} < \bar{t}_{S,2}$. Therefore, if the resource constraint (10) is satisfied for fragile sellers, it also holds for safe sellers. Because $T_S(\underline{s}, \bar{e}) \leq T_S(\underline{s}, \underline{e})$ it is sufficient that

$$T_S(\underline{s}, \underline{e}) \leq \bar{t}_{F,2} = \phi R(\underline{s}), \tag{A.25}$$

which, using the optimal transfer, is equivalent to

$$\pi \leq \phi R(\underline{s}). \tag{A.26}$$

□

Proof of Proposition 4. We consider an equilibrium in which the resource constraint (10) at $t = 2$ for fragile sellers is binding. Because $\phi_F < \phi_S$, safe sellers have access to more resources and, thus, the resource constraint is not binding for safe sellers. We assume that resources at $t = 1$ are sufficient such that there are no early defaults.

Risk sharing Consider the first-order conditions from the proof of Proposition 3. (A.19) then implies

$$\eta(\underline{s}, \underline{e}) - \xi \mathbb{P}(\underline{s}, \underline{e}) - \sigma_{F,2} = 0 \tag{A.27}$$

$$\text{and} \quad \eta(\tilde{s}, \tilde{e}) - \xi \mathbb{P}(\tilde{s}, \tilde{e}) = 0 \quad \forall (\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e}). \tag{A.28}$$

Therefore, combining with (A.18) yields

$$\xi = \frac{\eta(\tilde{s},\tilde{e})}{\mathbb{P}(\tilde{s},\tilde{e})} = u'(\tilde{s},\tilde{e}) \quad \forall(\tilde{s},\tilde{e}) \neq (\underline{s},\underline{e}) \tag{A.29}$$

and, thus,

$$\eta(\underline{s},\underline{e}) - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\underline{e}) - \sigma_{F,2} = 0 \tag{A.30}$$

$$\Leftrightarrow \sigma_{F,2} = \eta(\underline{s},\underline{e}) - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\underline{e}) \tag{A.31}$$

$$\Leftrightarrow \sigma_{F,2} = \mathbb{P}(\underline{s},\underline{e})(u'(\underline{s},\underline{e}) - u'(\bar{s},\bar{e})). \tag{A.32}$$

Due to $\sigma_{F,2} > 0$, it is $u'(\underline{s},\underline{e}) > u'(\bar{s},\bar{e})$ and, therefore, using the concavity of the utility function, $u(\underline{s},\underline{e}) < u(\bar{s},\bar{e})$.

Optimal margin Using the above in the first-order condition for α (A.20) gives

$$-\xi\mathbb{P}(\underline{s})(1-k)R(\underline{s}) - \sigma_{F,2}(1-k)R(\underline{s}) \tag{A.33}$$

$$= -\left[\xi\mathbb{P}(\underline{s}) + \sigma_{F,2}\right](1-k)R(\underline{s}) < 0, \tag{A.34}$$

which is strictly negative due to $\xi > 0$ and $\sigma_{F,2} > 0$. Thus, the optimal contract uses no margin.

Transfers The participation constraint (8) implies that $\mathbb{E}[T_S] = -\mathbb{P}(\underline{s})\alpha(1-k)R = 0$, using that $\alpha = 0$. Using the CCP's budget constraints (3), this implies that $\mathbb{E}[T_B] = 0$. Due to the binding resource constraint (10), the transfer in $(\underline{s},\underline{e})$ is given by

$$T_S(\underline{s},\underline{e}) = \phi R(\underline{s}) = \bar{t}_{F,2}. \tag{A.35}$$

(A.29) implies that

$$1 - T_B(\bar{s},\bar{e}) = 1 - T_B(\underline{s},\bar{e}) = -T_B(\bar{s},\underline{e}), \tag{A.36}$$

which, using the budget constraints (3) implies that

$$1 + T_S(\bar{s},\bar{e}) = 1 + T_S(\underline{s},\bar{e}) = T_S(\bar{s},\underline{e}). \tag{A.37}$$

Therefore,

$$\mathbb{E}[T_S \mid \bar{s}] = \mathbb{P}(\bar{e} \mid \bar{s})T_S(\bar{s},\bar{e}) + \mathbb{P}(\underline{e} \mid \bar{s})T_S(\bar{s},\underline{e}) \tag{A.38}$$

$$= \bar{\pi}(-1 + T_S(\bar{s},\underline{e})) + (1-\bar{\pi})T_S(\bar{s},\underline{e}) \tag{A.39}$$

$$= T_S(\bar{s},\underline{e}) - \bar{\pi}. \tag{A.40}$$

The participation constraint (8) then implies that

$$
\begin{aligned}
0 &= \mathbb{E}[T_S] && \text{(A.41)}\\
&= \mathbb{P}(\bar{s})\mathbb{E}[T_S \mid \bar{s}] + \mathbb{P}(\underline{s})\mathbb{E}[T_S \mid \underline{s}] && \text{(A.42)}\\
&= \mathbb{P}(\bar{s})(T_S(\bar{s},\underline{e}) - \overline{\pi}) + \mathbb{P}(\underline{s},\underline{e})T_S(\underline{s},\underline{e}) + \mathbb{P}(\underline{s},\bar{e})T_S(\underline{s},\bar{e}) && \text{(A.43)}\\
&= \mathbb{P}(\bar{s})(T_S(\bar{s},\underline{e}) - \overline{\pi}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2} + \mathbb{P}(\underline{s},\bar{e})(T_S(\bar{s},\underline{e}) - 1) && \text{(A.44)}\\
&= (1 - \mathbb{P}(\underline{s},\underline{e}))T_S(\bar{s},\underline{e}) - \mathbb{P}(\bar{s},\bar{e}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2} - \mathbb{P}(\underline{s},\bar{e}) && \text{(A.45)}\\
&= (1 - \mathbb{P}(\underline{s},\underline{e}))T_S(\bar{s},\underline{e}) - \mathbb{P}(\bar{e}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2} && \text{(A.46)}\\
\Leftrightarrow T_S(\bar{s},\underline{e}) &= \frac{\pi - \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2}}{1 - \mathbb{P}(\underline{s},\underline{e})}. && \text{(A.47)}
\end{aligned}
$$

Together with (A.37), this yields

$$
\begin{aligned}
T_S(\underline{s},\bar{e}) = T_S(\bar{s},\bar{e}) &= T_S(\bar{s},\underline{e}) - 1 && \text{(A.48)}\\
&= \frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2}}{1 - \mathbb{P}(\underline{s},\underline{e})} - 1 && \text{(A.49)}\\
&= \frac{-\mathbb{P}(\bar{s},\underline{e}) - \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2}}{1 - \mathbb{P}(\underline{s},\underline{e})}. && \text{(A.50)}
\end{aligned}
$$

No defaults at $t = 1$ Because $\alpha = 0$, there are no defaults at $t = 1$. □

A.2.2 With Defaults

Proof of Proposition 5. If fragile sellers default in $(\underline{s},\underline{e})$, the optimal contract maximizes expected buyer utility (4) subject to the CCP's budget constraints (3) and (13), the participation constraint (15), the resource constraints at $t = 1$ (9), and the resource constraint of safe sellers at $t = 2$ (10). The derivatives of the Lagrangian with respect to $T_B(\tilde{s},\tilde{e})$, $T_S(\tilde{s},\tilde{e})$, and α, respectively, are

$$
\begin{aligned}
&(\partial T_B(\tilde{s},\tilde{e})) \;\; -\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e}) + \eta(\tilde{s},\tilde{e}) \quad \forall(\tilde{s},\tilde{e}) && \text{(A.51)}\\
&(\partial T_S(\underline{s},\underline{e})) \;\; (1-\gamma)\eta(\underline{s},\underline{e}) - \xi\mathbb{P}(\underline{s},\underline{e})(1-\gamma) - \sigma_{S,2} && \text{(A.52)}\\
&(\partial T_S(\tilde{s},\tilde{e})) \;\; \eta(\tilde{s},\tilde{e}) - \xi\mathbb{P}(\tilde{s},\tilde{e}) \quad \forall(\tilde{s},\tilde{e}) \neq (\underline{s},\underline{e}) && \text{(A.53)}\\
&\quad (\partial\alpha) \;\; \xi\left(-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s},\underline{e})\gamma(1-k)\right)R(\underline{s}) + \eta(\underline{s},\underline{e})\gamma(k-\rho)R(\underline{s})\\
&\qquad - \sigma_{S,2}(1-k)R(\underline{s}) - (\sigma_{1,F} + \sigma_{1,S})R(\underline{s}). && \text{(A.54)}
\end{aligned}
$$

Here, η is the Lagrange multiplier on the budget constraint, $\sigma_{j,t}$ that on the resource constraint for type $j \in \{S, F\}$ at time t, and ξ that on the participation constraint.

Risk sharing Suppose first that the resource constraint of safe sellers at $t = 2$ is slack, $\sigma_{S,2} = 0$. The first-order conditions (A.51), (A.52), and (A.53) then jointly imply that

$$\xi = \frac{\eta(\tilde{s}, \tilde{e})}{\mathbb{P}(\tilde{s}, \tilde{e})} = u'(\tilde{s}, \tilde{e}) \quad \forall(\tilde{s}, \tilde{e}). \tag{A.55}$$

Therefore, buyers are fully insured. Below, we characterize the condition under which the resource constraint of safe sellers is satisfied.

Transfers and markup Consider an arbitrary margin $\alpha \in [0, \phi]$. Taking expectations of the CCP's budget constraints gives

$$0 = \mathbb{E}[T_B] + \mathbb{E}[T_S] + \gamma\mathbb{P}(\underline{s}, \underline{e}) \left([\alpha k + \rho(\phi - \alpha)] R(\underline{s}) - T_S(\underline{s}, \underline{e}) \right), \tag{A.56}$$

or, equivalently,

$$\mathbb{E}[T_B] = -\mathbb{E}[T_S] + \gamma\mathbb{P}(\underline{s}, \underline{e})T_S(\underline{s}, \underline{e}) - \gamma\mathbb{P}(\underline{s}, \underline{e}) [\alpha k + \rho(\phi - \alpha)] R(\underline{s}). \tag{A.57}$$

The binding participation constraint (15) implies that

$$\begin{aligned} -\mathbb{E}[T_S] + \gamma\mathbb{P}(\underline{s}, \underline{e})T_S(\underline{s}, \underline{e}) = \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) \\ + \gamma\mathbb{P}(\underline{s}, \underline{e}) [\phi - (1-k)\alpha] R(\underline{s}). \end{aligned} \tag{A.58}$$

Combining these expressions yields

$$\begin{aligned} \mathbb{E}[T_B] = &\ [\mathbb{P}(\underline{s})(1-k)\alpha + \gamma\mathbb{P}(\underline{s}, \underline{e})(1-\rho)(\phi - \alpha)] R(\underline{s}) \\ =:&\ m^{NR}(\alpha). \end{aligned} \tag{A.59}$$

Thus, the markup compensates sellers for the opportunity cost of posting margin and for the deadweight cost associated with the uncollateralized assets of fragile sellers that are lost upon default.

Full insurance implies that

$$\tilde{e} - T_B(\tilde{s}, \tilde{e}) = \mathbb{E}[\tilde{e} - T_B] = \pi - m^{NR}(\alpha). \tag{A.60}$$

Hence, in states without default,

$$T_S(\tilde{s}, \bar{e}) = -(1-\pi) - m^{NR}(\alpha), \tag{A.61}$$

$$T_S(\bar{s}, \underline{e}) = \pi - m^{NR}(\alpha). \tag{A.62}$$

In state $(\underline{s}, \underline{e})$, the CCP's budget constraint gives

$$(1-\gamma)T_S(\underline{s},\underline{e}) + \gamma\left[\alpha k + \rho(\phi - \alpha)\right]R(\underline{s}) = \pi - m^{NR}(\alpha) \tag{A.63}$$

$$\Leftrightarrow \quad T_S(\underline{s},\underline{e}) = \frac{\pi - m^{NR}(\alpha) - \gamma\left[\alpha k + \rho(\phi-\alpha)\right]R(\underline{s})}{1-\gamma}. \tag{A.64}$$

Optimal margin Under full insurance, buyer utility is

$$u\left(\pi - m^{NR}(\alpha)\right). \tag{A.65}$$

Thus, the optimal margin minimizes $m^{NR}(\alpha)$ subject to $\alpha \in [0, \phi]$, where the upper bound ensures that fragile sellers do not default at $t = 1$. It is

$$\frac{\partial m^{NR}(\alpha)}{\partial \alpha} = \left[\mathbb{P}(\underline{s})(1-k) - \gamma\mathbb{P}(\underline{s},\underline{e})(1-\rho)\right]R(\underline{s}). \tag{A.66}$$

Therefore, if

$$\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho) < \mathbb{P}(\underline{s})(1-k), \tag{A.67}$$

the markup is increasing in α and the optimal margin is

$$\alpha^{NR} = 0. \tag{A.68}$$

Otherwise, the markup is weakly decreasing in α, and the optimal margin is

$$\alpha^{NR} = \phi. \tag{A.69}$$

If the condition holds with equality, all $\alpha \in [0, \phi]$ generate the same markup, and we select $\alpha^{NR} = \phi$ without loss of generality. In the following, let

$$m^{NR} := m^{NR}(\alpha^{NR}). \tag{A.70}$$

(No) Defaults at $t = 2$ Safe sellers do not default at $t = 2$ if and only if

$$T_S(\underline{s},\underline{e}) \leq \left[1-(1-k)\alpha^{NR}\right]R(\underline{s}) \tag{A.71}$$

$$\Leftrightarrow \quad \frac{\pi - m^{NR} - \gamma\left[\alpha^{NR}k + \rho(\phi-\alpha^{NR})\right]R(\underline{s})}{1-\gamma} \leq \left[1-(1-k)\alpha^{NR}\right]R(\underline{s}) \tag{A.72}$$

$$\Leftrightarrow \quad \gamma \leq \frac{R(\underline{s}) - \pi - \mathbb{P}(\bar{s})(1-k)\alpha^{NR}R(\underline{s})}{R(\underline{s})\left(1-\left[\mathbb{P}(\underline{s},\underline{e})(1-\rho)+\rho\right]\phi - (1-\rho)\left[1-\mathbb{P}(\underline{s},\underline{e})\right]\alpha^{NR}\right)} =: \overline{\gamma}^{NR}\left(\alpha^{NR}\right). \tag{A.73}$$

The denominator is strictly positive because $\alpha^{NR} \leq \phi$ implies

$$[\mathbb{P}(\underline{s},\underline{e})(1-\rho)+\rho]\,\phi+(1-\rho)\,[1-\mathbb{P}(\underline{s},\underline{e})]\,\alpha^{NR} \tag{A.74}$$

$$\leq [\mathbb{P}(\underline{s},\underline{e})(1-\rho)+\rho+(1-\rho)(1-\mathbb{P}(\underline{s},\underline{e}))]\,\phi=\phi<1. \tag{A.75}$$

Thus, for $\gamma < \overline{\gamma}^{NR}(\alpha^{NR})$, the resource constraint of safe sellers is slack, confirming the full-insurance allocation derived above. At equality, the resource constraint binds but the full-insurance allocation remains feasible.

Fragile sellers' resources at $t=2$ are

$$\bar{t}_{F,2}=\left[\phi-(1-k)\alpha^{NR}\right]R(\underline{s}). \tag{A.76}$$

Using the transfer and markup derived above,

$$(1-\gamma)\,[T_S(\underline{s},\underline{e})-\bar{t}_{F,2}] \tag{A.77}$$

$$=\pi-\left[\phi-\mathbb{P}(\bar{s})(1-k)\alpha^{NR}-\gamma(1-\rho)\,[1-\mathbb{P}(\underline{s},\underline{e})]\left(\phi-\alpha^{NR}\right)\right]R(\underline{s}) \tag{A.78}$$

$$\geq \pi-\phi R(\underline{s})>0, \tag{A.79}$$

where the last inequality follows from Assumption (1). Therefore, fragile sellers default in state $(\underline{s},\underline{e})$. Because

$$T_S(\underline{s},\bar{e})=-(1-\pi)-m^{NR}<0, \tag{A.80}$$

they do not default in state $(\underline{s},\bar{e})$.

No defaults at $t=1$ Because $\alpha^{NR} \in \{0,\phi\}$, it is $\alpha^{NR} \leq \phi$, so fragile sellers can meet the margin call at $t=1$. Since $\phi<1$, safe sellers can meet it as well. □

Proof of Proposition 6. Consider an interior solution for T_S, T_B, and α, such that the first-order conditions (A.51) to (A.54) are all equal to zero. Due to Assumption (2), safe sellers' resource constraint (10) is binding in equilibrium (otherwise, full insurance with the optimal margin α^{NR} would be feasible, contradicting the assumption) and, thus, $T_S(\underline{s},\underline{e})=R(\underline{s})-(1-k)\alpha R(\underline{s})$ and $\sigma_{S,2}>0$.

Risk sharing (A.53) together with (A.51) imply that

$$\xi=\frac{\eta(\tilde{s},\tilde{e})}{\mathbb{P}(\tilde{s},\tilde{e})}=u'(\tilde{s},\tilde{e}) \quad \forall(\tilde{s},\tilde{e})\neq(\underline{s},\underline{e}), \tag{A.81}$$

which implies that $u(\tilde{s},\tilde{e}) = u(\bar{s},\bar{e})$ for all states $(\tilde{s},\tilde{e}) \neq (\underline{s},\underline{e})$. (A.52) gives

$$
\begin{aligned}
&(1-\gamma)\eta(\underline{s},\underline{e}) - \xi\mathbb{P}(\underline{s},\underline{e})(1-\gamma) = \sigma_{S,2} && \text{(A.82)}\\
\Leftrightarrow &(1-\gamma)(\mathbb{P}(\underline{s},\underline{e})u'(\underline{s},\underline{e}) - u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\underline{e})) = \sigma_{S,2} && \text{(A.83)}\\
\Leftrightarrow &(1-\gamma)\mathbb{P}(\underline{s},\underline{e})(u'(\underline{s},\underline{e}) - u'(\underline{s},\bar{e})) = \sigma_{S,2}, && \text{(A.84)}
\end{aligned}
$$

and, thus, using the concavity of $u(\cdot)$, it is $u(\underline{s},\underline{e}) < u(\underline{s},\bar{e}) = u(\bar{s},\underline{e}) = u(\bar{s},\bar{e})$.

Transfers The participation constraint (15) is binding in equilibrium, which implies that

$$
\begin{aligned}
&0 = -\mathbb{E}[T_S] - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) + \gamma\mathbb{P}(\underline{s},\underline{e})\left(T_S(\underline{s},\underline{e}) + ((1-k)\alpha - \phi)R(\underline{s})\right) && \text{(A.85)}\\
\Leftrightarrow &\mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) - \gamma\mathbb{P}(\underline{s},\underline{e})((1-k)\alpha - \phi)R(\underline{s}) = -\mathbb{E}[T_S] + \gamma\mathbb{P}(\underline{s},\underline{e})T_S(\underline{s},\underline{e}), && \text{(A.86)}
\end{aligned}
$$

which together with the budget constraints (3) and (13) implies that

$$
\begin{aligned}
&\mathbb{E}[T_B] + \mathbb{E}[T_S] - \mathbb{P}(\underline{s},\underline{e})\gamma\left(T_S(\underline{s},\underline{e}) - \alpha k R(\underline{s}) - \rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right) = 0 && \text{(A.87)}\\
\Leftrightarrow &\mathbb{E}[T_B] = -\mathbb{E}[T_S] + \gamma\mathbb{P}(\underline{s},\underline{e})\left(T_S(\underline{s},\underline{e}) - \alpha k R(\underline{s}) - \rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right) && \text{(A.88)}\\
\Leftrightarrow &\mathbb{E}[T_B] = \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) - \gamma\mathbb{P}(\underline{s},\underline{e})((1-k)\alpha - \phi)R(\underline{s}) - \gamma\mathbb{P}(\underline{s},\underline{e})\left(\alpha k R(\underline{s}) + \rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right)\\
\Leftrightarrow &\mathbb{E}[T_B] = \left[(\mathbb{P}(\underline{s}) - \gamma\mathbb{P}(\underline{s},\underline{e}))(1-k)\alpha + \gamma\mathbb{P}(\underline{s},\underline{e})\left(\phi - \alpha k - \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)\right]R(\underline{s}) =: m, && \text{(A.89)}
\end{aligned}
$$

i.e., the price of the contract is not actuarially fair but there is a markup that compensates for the cost of posting margin and for the deadweight cost of defaults. This markup is equal to

$$
m = \left(\mathbb{P}(\underline{s})(1-k)\alpha + \gamma\mathbb{P}(\underline{s},\underline{e})\left(\phi - \alpha - \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)\right)R(\underline{s}). \tag{A.90}
$$

The transfer in $(\underline{s},\underline{e})$ is determined by the binding resource constraint (10):

$$
T_S(\underline{s},\underline{e}) = (1-\alpha)R(\underline{s}) + \alpha k R(\underline{s}) = \bar{t}_{S,2}. \tag{A.91}
$$

The result on risk sharing above implies that

$$
1 - T_B(\bar{s},\bar{e}) = 1 - T_B(\underline{s},\bar{e}) = -T_B(\bar{s},\underline{e}), \tag{A.92}
$$

which, using the budget constraints (3) implies that

$$
1 + T_S(\bar{s},\bar{e}) = 1 + T_S(\underline{s},\bar{e}) = T_S(\bar{s},\underline{e}). \tag{A.93}
$$

Therefore,

$$\mathbb{E}[T_S \mid \bar{s}] = \mathbb{P}(\bar{e} \mid \bar{s}) T_S(\bar{s}, \bar{e}) + \mathbb{P}(\underline{e} \mid \bar{s}) T_S(\bar{s}, \underline{e}) \tag{A.94}$$

$$= \bar{\pi}(-1 + T_S(\bar{s}, \underline{e})) + (1 - \bar{\pi}) T_S(\bar{s}, \underline{e}) \tag{A.95}$$

$$= T_S(\bar{s}, \underline{e}) - \bar{\pi}. \tag{A.96}$$

The participation constraint (15) then implies that

$$\gamma \mathbb{P}(\underline{s}, \underline{e}) T_S(\underline{s}, \underline{e}) - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) + \gamma \mathbb{P}(\underline{s}, \underline{e})((1-k)\alpha - \phi) R(\underline{s}) - m + \left[(\mathbb{P}(\underline{s}) - \gamma \mathbb{P}(\underline{s}, \underline{e}))(1-k)\alpha + \gamma \mathbb{P}(\underline{s}, \underline{e}) \left(\phi - \alpha k - \rho \left(1 - \frac{\alpha}{\phi}\right) \phi \right) \right] R(\underline{s}) = \mathbb{E}[T_S] \tag{A.97}$$

$$\Leftrightarrow \gamma \mathbb{P}(\underline{s}, \underline{e}) \left(T_S(\underline{s}, \underline{e}) - \alpha k R(\underline{s}) - \rho \left(1 - \frac{\alpha}{\phi}\right) \phi R(\underline{s}) \right) - m = \mathbb{E}[T_S] \tag{A.98}$$

$$= \mathbb{P}(\bar{s}) \mathbb{E}[T_S \mid \bar{s}] + \mathbb{P}(\underline{s}) \mathbb{E}[T_S \mid \underline{s}] \tag{A.99}$$

$$= \mathbb{P}(\bar{s})(T_S(\bar{s}, \underline{e}) - \bar{\pi}) + \mathbb{P}(\underline{s}, \underline{e}) T_S(\underline{s}, \underline{e}) + \mathbb{P}(\underline{s}, \bar{e}) T_S(\underline{s}, \bar{e}) \tag{A.100}$$

$$= \mathbb{P}(\bar{s})(T_S(\bar{s}, \underline{e}) - \bar{\pi}) + \mathbb{P}(\underline{s}, \underline{e}) \bar{t}_{S,2} + \mathbb{P}(\underline{s}, \bar{e})(T_S(\bar{s}, \underline{e}) - 1) \tag{A.101}$$

$$= (1 - \mathbb{P}(\underline{s}, \underline{e})) T_S(\bar{s}, \underline{e}) - \mathbb{P}(\bar{s}, \bar{e}) + \mathbb{P}(\underline{s}, \underline{e}) \bar{t}_{S,2} - \mathbb{P}(\underline{s}, \bar{e}) \tag{A.102}$$

$$= (1 - \mathbb{P}(\underline{s}, \underline{e})) T_S(\bar{s}, \underline{e}) - \mathbb{P}(\bar{e}) + \mathbb{P}(\underline{s}, \underline{e}) \bar{t}_{S,2} \tag{A.103}$$

$$\Leftrightarrow T_S(\bar{s}, \underline{e}) = \frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s}, \underline{e}) \bar{t}_{S,2} - m + \mathbb{P}(\underline{s}, \underline{e}) \gamma \left(\bar{t}_{S,2} - \alpha k R(\underline{s}) - \rho \left(1 - \frac{\alpha}{\phi}\right) \phi R(\underline{s}) \right)}{1 - \mathbb{P}(\underline{s}, \underline{e})} \tag{A.104}$$

$$\Leftrightarrow T_S(\bar{s}, \underline{e}) = \frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s}, \underline{e})((1-\gamma) \bar{t}_{S,2} + \gamma(\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi) R(\underline{s})) - m}{1 - \mathbb{P}(\underline{s}, \underline{e})}. \tag{A.105}$$

Together with the result on risk sharing, this yields

$$T_S(\underline{s}, \bar{e}) = T_S(\bar{s}, \bar{e}) = T_S(\bar{s}, \underline{e}) - 1 \tag{A.106}$$

$$= \frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s}, \underline{e})((1-\gamma) \bar{t}_{S,2} + \gamma(\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi) R(\underline{s})) - m}{1 - \mathbb{P}(\underline{s}, \underline{e})} - 1 \tag{A.107}$$

$$= \frac{-\mathbb{P}(\bar{s}, \underline{e}) - \mathbb{P}(\underline{s}, \underline{e})((1-\gamma) \bar{t}_{S,2} + \gamma(\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi) R(\underline{s})) - m}{1 - \mathbb{P}(\underline{s}, \underline{e})}. \tag{A.108}$$

Buyer transfers. In all non-default states, the CCP budget constraint implies

$$T_B(\tilde{s}, \tilde{e}) = -T_S(\tilde{s}, \tilde{e}), \qquad (\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e}). \tag{A.109}$$

In the default state $(\underline{s}, \underline{e})$, the CCP budget is

$$T_B(\underline{s}, \underline{e}) + (1-\gamma) T_S(\underline{s}, \underline{e}) + \gamma \left[\alpha k R(\underline{s}) + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi R(\underline{s}) \right] = 0. \tag{A.110}$$

Using $T_S(\underline{s}, \underline{e}) = \bar{t}_{S,2}$, this yields

$$T_B(\underline{s}, \underline{e}) = -(1-\gamma)\bar{t}_{S,2} - \gamma \left[\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi \right] R(\underline{s}). \tag{A.111}$$

For the state $(\bar{s}, \underline{e})$, we have

$$\begin{aligned} T_B(\bar{s}, \underline{e}) &= -T_S(\bar{s}, \underline{e}) && \text{(A.112)} \\ &= -\frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s}, \underline{e}) \left((1-\gamma)\bar{t}_{S,2} + \gamma \left(\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi \right) R(\underline{s}) \right) - m}{1 - \mathbb{P}(\underline{s}, \underline{e})} && \text{(A.113)} \\ &= \frac{-\mathbb{P}(\bar{e}) + \mathbb{P}(\underline{s}, \underline{e}) \left((1-\gamma)\bar{t}_{S,2} + \gamma \left(\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi \right) R(\underline{s}) \right) + m}{1 - \mathbb{P}(\underline{s}, \underline{e})}. && \text{(A.114)} \end{aligned}$$

Since risk sharing implies

$$1 - T_B(\bar{s}, \bar{e}) = 1 - T_B(\underline{s}, \bar{e}) = -T_B(\bar{s}, \underline{e}), \tag{A.115}$$

we obtain

$$\begin{aligned} T_B(\bar{s}, \bar{e}) = T_B(\underline{s}, \bar{e}) &= 1 + T_B(\bar{s}, \underline{e}) && \text{(A.116)} \\ &= 1 + \frac{-\mathbb{P}(\bar{e}) + \mathbb{P}(\underline{s}, \underline{e}) \left((1-\gamma)\bar{t}_{S,2} + \gamma \left(\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi \right) R(\underline{s}) \right) + m}{1 - \mathbb{P}(\underline{s}, \underline{e})} && \text{(A.117)} \\ &= \frac{\mathbb{P}(\bar{s}, \underline{e}) + \mathbb{P}(\underline{s}, \underline{e}) \left((1-\gamma)\bar{t}_{S,2} + \gamma \left(\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi \right) R(\underline{s}) \right) + m}{1 - \mathbb{P}(\underline{s}, \underline{e})}. && \text{(A.118)} \end{aligned}$$

Therefore,

$$T_B(\underline{s}, \bar{e}) = T_B(\bar{s}, \bar{e}) = \frac{\mathbb{P}(\bar{s}, \underline{e}) + \mathbb{P}(\underline{s}, \underline{e}) \left((1-\gamma)\bar{t}_{S,2} + \gamma \left(\alpha k + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi \right) R(\underline{s}) \right) + m}{1 - \mathbb{P}(\underline{s}, \underline{e})}. \tag{A.119}$$

Optimal margin The derivative of the Lagrangian with respect to α is given by (A.54), which is equal to

$$\begin{aligned}
&\xi\left(-\mathbb{P}(\underline{s})(1-k)+\mathbb{P}(\underline{s},\underline{e})\gamma(1-k)\right)R(\underline{s})+\eta(\underline{s},\underline{e})\gamma(k-\rho)R(\underline{s})-\sigma_{S,2}(1-k)R(\underline{s}) \quad \text{(A.120)}\\
=&u'(\underline{s},\bar{e})\left(-\mathbb{P}(\underline{s})(1-k)+\mathbb{P}(\underline{s},\underline{e})\gamma(1-k)\right)R(\underline{s})+\mathbb{P}(\underline{s},\underline{e})u'(\underline{s},\underline{e})\gamma(k-\rho)R(\underline{s})-\sigma_{S,2}(1-k)R(\underline{s})\\
=&u'(\underline{s},\bar{e})\left(-\mathbb{P}(\underline{s})(1-k)+\mathbb{P}(\underline{s},\underline{e})\gamma(1-k)\right)R(\underline{s})+\mathbb{P}(\underline{s},\underline{e})u'(\underline{s},\underline{e})\gamma(k-\rho)R(\underline{s})-\sigma_{S,2}(1-k)R(\underline{s})\\
=&u'(\underline{s},\bar{e})\left(\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho)-\mathbb{P}(\underline{s})(1-k)\right)R(\underline{s})+\mathbb{P}(\underline{s},\underline{e})(u'(\underline{s},\underline{e})-u'(\underline{s},\bar{e}))(\gamma(k-\rho)-(1-\gamma)(1-k))R(\underline{s})
\end{aligned}$$

and, thus, an interior optimum for the margin requirement is reached if and only if

$$\begin{aligned}
u'(\underline{s},\bar{e})\mathbb{P}(\underline{s})(1-k)=&u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho)\\
&+(u'(\underline{s},\underline{e})-u'(\underline{s},\bar{e}))\mathbb{P}(\underline{s},\underline{e})(\gamma(k-\rho)-(1-\gamma)(1-k)). \quad \text{(A.121)}
\end{aligned}$$

(No) Defaults at $t=2$ Safe sellers do not default at $t=2$ as their resource constraint (10) is satisfied with equality. Fragile sellers default in state $(\underline{s},\underline{e})$ at $t=2$ if and only if (using the binding resource constraint of safe sellers at $t=2$)

$$\begin{aligned}
&T_S(\underline{s},\underline{e})>\phi R(\underline{s})-(1-k)\alpha R(\underline{s}) && \text{(A.122)}\\
\Leftrightarrow &R(\underline{s})-(1-k)\alpha R(\underline{s})>\phi R(\underline{s})-(1-k)\alpha R(\underline{s}) && \text{(A.123)}\\
\Leftrightarrow &R(\underline{s})>\phi R(\underline{s}) && \text{(A.124)}\\
\Leftrightarrow &1>\phi, && \text{(A.125)}
\end{aligned}$$

which holds by assumption.

No defaults at $t=1$ Using the resource constraint (9), fragile sellers do not default at $t=1$ if

$$\alpha\leq\phi, \quad \text{(A.126)}$$

which, due to $\phi<1$, is sufficient to ensure no defaults of safe sellers. □

Proof of Proposition 7. Consider the set of resource-compatible contracts with defaults of fragile sellers at $t=2$ in state $(\underline{s},\underline{e})$ and suppose that Assumption 2 holds. If $\alpha=0$, then the markup is $m=\gamma\mathbb{P}(\underline{s},\underline{e})(1-\rho)\phi R(\underline{s})$ and the transfer in $(\underline{s},\underline{e})$ is equal to $T_S(\underline{s},\underline{e})=R(\underline{s})-m$, which provides buyers with the following consumption:

$$\begin{aligned}
0-T_B(\underline{s},\underline{e})&=(1-\gamma)T_S(\underline{s},\underline{e})+\gamma\rho\phi R(\underline{s}) && \text{(A.127)}\\
&=(1-\gamma)(R(\underline{s})-m)+\gamma\rho\phi R(\underline{s}) && \text{(A.128)}\\
&=(1-\gamma)R(\underline{s})-\gamma((1-\gamma)\mathbb{P}(\underline{s},\underline{e})(1-\rho)-\rho)\phi R(\underline{s}). && \text{(A.129)}
\end{aligned}$$

In contrast, the optimal contract without defaults has $\alpha = 0$, $m = 0$, and $T_S(\underline{s}, \underline{e}) = \phi R(\underline{s})$, which provides buyers with the following consumption:

$$0 - T_B(\underline{s}, \underline{e}) = T_S(\underline{s}, \underline{e}) \tag{A.130}$$
$$= \phi R(\underline{s}). \tag{A.131}$$

Both contracts provide partial insurance with $u(\underline{s}, \underline{e}) < u(\tilde{s}, \tilde{e})$ for all $(\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e})$. Consider the case that $\phi = 0$. Then, the markup for both contracts is $m = 0$. Therefore, the contract with defaults provides more risk sharing if and only if

$$(1 - \gamma) R(\underline{s}) > 0, \tag{A.132}$$

which holds due to $\gamma < 1$. The result follows from continuity in ϕ. □

A.3 Central Clearing with Replacement

A.3.1 Full Insurance

Proof of Proposition 8. The budget constraints (3) and (21) imply that

$$\mathbb{E}[T_B] + \mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\rho\phi R(\underline{s}) - \mathbb{P}(\underline{s})\gamma(\mathbb{E}[T_S \mid \underline{s}] + C) = 0 \tag{A.133}$$
$$\Leftrightarrow \mathbb{E}[T_B] - \mathbb{P}(\underline{s})\gamma C + \mathbb{P}(\underline{s})\gamma\rho\phi R(\underline{s}) = -\mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\mathbb{E}[T_S \mid \underline{s}]. \tag{A.134}$$

Using this in the binding participation constraint (23) yields

$$0 = -\mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\mathbb{E}[T_S \mid \underline{s}] - \mathbb{P}(\underline{s})\gamma\phi R(\underline{s}) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s}) \tag{A.135}$$
$$\Leftrightarrow 0 = \mathbb{E}[T_B] - \mathbb{P}(\underline{s})\gamma C + \mathbb{P}(\underline{s})\gamma\rho\phi R(\underline{s}) - \mathbb{P}(\underline{s})\gamma\phi R(\underline{s}) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s})$$
$$\Leftrightarrow 0 = \mathbb{E}[T_B] - \mathbb{P}(\underline{s})\gamma C - \mathbb{P}(\underline{s})\gamma(1-\rho)\phi R(\underline{s}) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s})$$
$$\Leftrightarrow \mathbb{E}[T_B] = \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\alpha R(\underline{s})) =: m, \tag{A.136}$$

where m is the markup of the contract.

Full insurance implies that $\tilde{e} - T_B(\tilde{s}, \tilde{e}) \equiv \mathbb{E}[\tilde{e} - T_B(\tilde{s}, \tilde{e})]$, where the right hand side is equal to $\pi \cdot 1 + (1 - \pi) \cdot 0 - m$. Thus,

$$1 - T_B(\tilde{s}, \bar{e}) = \pi - m \tag{A.137}$$
$$\text{and } 0 - T_B(\tilde{s}, \underline{e}) = \pi - m. \tag{A.138}$$

Using the CCP's budget constraints, this implies that

$$1 + T_S(\bar{s}, \bar{e}) = \pi - m \tag{A.139}$$
$$0 + T_S(\bar{s}, \underline{e}) = \pi - m \tag{A.140}$$
$$1 + T_S(\underline{s}, \bar{e}) - \gamma(\mathbb{E}[T_S \mid \underline{s}] + C - \rho\phi R(\underline{s})) = \pi - m \tag{A.141}$$
$$T_S(\underline{s}, \underline{e}) - \gamma(\mathbb{E}[T_S \mid \underline{s}] + C - \rho\phi R(\underline{s})) = \pi - m. \tag{A.142}$$

Taking the $\mathbb{P}(\cdot \mid \underline{s})$-weighted sum of the last two equations yields

$$\mathbb{P}(\bar{e} \mid \underline{s}) + \mathbb{E}[T_S \mid \underline{s}] - \gamma(\mathbb{E}[T_S \mid \underline{s}] + C - \rho\phi R(\underline{s})) = \pi - m \tag{A.143}$$
$$\Leftrightarrow \mathbb{P}(\bar{e} \mid \underline{s}) + (1-\gamma)\mathbb{E}[T_S \mid \underline{s}] = \pi - m + \gamma(C - \rho\phi R(\underline{s})) \tag{A.144}$$
$$\Leftrightarrow \mathbb{E}[T_S \mid \underline{s}] = \frac{\pi - \underline{\pi} - m + \gamma(C - \rho\phi R(\underline{s}))}{1-\gamma}. \tag{A.145}$$

Therefore, transfers are given by

$$T_S(\bar{s}, \bar{e}) = -(1-\pi) - m \tag{A.146}$$
$$T_S(\bar{s}, \underline{e}) = \pi - m \tag{A.147}$$
$$T_S(\underline{s}, \bar{e}) = -(1-\pi) + \gamma\left(\frac{\pi - \underline{\pi} - m + \gamma(C - \rho\phi R(\underline{s}))}{1-\gamma} + C - \rho\phi R(\underline{s})\right) - m \tag{A.148}$$
$$= -(1-\pi) + \gamma\frac{\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})}{1-\gamma} - m \tag{A.149}$$
$$T_S(\underline{s}, \underline{e}) = \pi + \gamma\frac{\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})}{1-\gamma} - m. \tag{A.150}$$

□

Proof of Proposition 9. The optimal contract maximizes expected buyer utility (4) subject to the CCP's budget constraints (3) and (21), and the participation constraint (23) and resource constraints of safe sellers (9) and (10) and that of fragile sellers (30). Therefore, the partial derivative of the Lagrangian with respect to $T_B(\tilde{s}, \tilde{e})$, $T_S(\tilde{s}, \tilde{e})$, and α, respectively, are

$$[\partial T_B(\tilde{s}, \tilde{e})] \quad -\mathbb{P}(\tilde{s}, \tilde{e})u'(\tilde{s}, \tilde{e}) + \eta(\tilde{s}, \tilde{e}) \tag{A.151}$$
$$[\partial T_S(\bar{s}, \tilde{e})] \quad \eta(\bar{s}, \tilde{e}) - \xi\mathbb{P}(\bar{s}, \tilde{e}) \tag{A.152}$$
$$[\partial T_S(\underline{s}, \underline{e})] \quad \eta(\underline{s}, \underline{e})(1 - \gamma\mathbb{P}(\underline{e} \mid \underline{s})) - \eta(\underline{s}, \bar{e})\gamma\mathbb{P}(\underline{e} \mid \underline{s}) - \xi\mathbb{P}(\underline{s}, \underline{e})(1-\gamma) - \sigma_{S,2}(\underline{s}, \underline{e}) \tag{A.153}$$
$$[\partial T_S(\underline{s}, \bar{e})] \quad \eta(\underline{s}, \bar{e})(1 - \gamma\mathbb{P}(\bar{e} \mid \underline{s})) - \eta(\underline{s}, \underline{e})\gamma\mathbb{P}(\bar{e} \mid \underline{s}) - \xi\mathbb{P}(\underline{s}, \bar{e})(1-\gamma) - \sigma_{S,2}(\underline{s}, \bar{e}) \tag{A.154}$$
$$[\partial\alpha] \quad -\xi\mathbb{P}(\underline{s})(1-\gamma)(1-k)R(\underline{s}) + \sigma_{F,1} - (\sigma_{S,2}(\underline{s}, \underline{e}) + \sigma_{S,2}(\underline{s}, \bar{e}))(1-k)R(\underline{s}), \tag{A.155}$$

An interior solution satisfies the conditions with equality. We assume that the resource constraints of safe sellers after a negative signal are not binding and, thus, $\sigma_{S,2}(\underline{s}, \tilde{e}) \equiv 0$.

Margin An interior solution for the margin has (A.155) equal to zero:

$$\xi\mathbb{P}(\underline{s})(1-\gamma)(1-k)R(\underline{s}) = \sigma_{F,1}, \tag{A.156}$$

which, given the binding participation constraint $\xi > 0$ implies that $\sigma_{F,1} > 0$. Therefore, the optimal margin is given by $\alpha^* = \phi$.

Risk sharing Combining (A.153) and (A.154) yields

$$\begin{aligned}
&\frac{1}{\mathbb{P}(\underline{s},\bar{e})}\Big(\eta(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s})) - \eta(\underline{s},\underline{e})\gamma\mathbb{P}(\bar{e}\mid\underline{s}) - \xi\mathbb{P}(\underline{s},\bar{e})(1-\gamma)\Big) \\
&- \frac{1}{\mathbb{P}(\underline{s},\underline{e})}\Big(\eta(\underline{s},\underline{e})(1-\gamma\mathbb{P}(\underline{e}\mid\underline{s})) - \eta(\underline{s},\bar{e})\gamma\mathbb{P}(\underline{e}\mid\underline{s}) - \xi\mathbb{P}(\underline{s},\underline{e})(1-\gamma)\Big) && \text{(A.157)}\\
=&\frac{\eta(\underline{s},\bar{e})}{\mathbb{P}(\underline{s},\bar{e})} - \gamma\frac{\eta(\underline{s},\underline{e})+\eta(\underline{s},\bar{e})}{\mathbb{P}(\underline{s})} - \xi(1-\gamma) - \left(\frac{\eta(\underline{s},\underline{e})}{\mathbb{P}(\underline{s},\underline{e})} - \gamma\frac{\eta(\underline{s},\underline{e})+\eta(\underline{s},\bar{e})}{\mathbb{P}(\underline{s})} - \xi(1-\gamma)\right) && \text{(A.158)}\\
=&\frac{\eta(\underline{s},\bar{e})}{\mathbb{P}(\underline{s},\bar{e})} - \frac{\eta(\underline{s},\underline{e})}{\mathbb{P}(\underline{s},\underline{e})} && \text{(A.159)}\\
=&u'(\underline{s},\bar{e}) - u'(\underline{s},\underline{e}) = 0, && \text{(A.160)}
\end{aligned}$$

where in the last step we use (A.151). Therefore, $u'(\underline{s},\bar{e}) = u'(\underline{s},\underline{e})$. (A.152) together with (A.151) imply that $\xi = u'(\bar{s},\underline{e}) = u'(\bar{s},\bar{e})$. Using this in (A.154) and setting it to zero yields

$$\begin{aligned}
&u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s})) - u'(\underline{s},\underline{e})\mathbb{P}(\underline{s},\underline{e})\gamma\mathbb{P}(\bar{e}\mid\underline{s}) - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) && \text{(A.161)}\\
&= u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s})) - u'(\underline{s},\bar{e})\gamma\mathbb{P}(\underline{s},\underline{e})\mathbb{P}(\bar{e}\mid\underline{s}) - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) && \text{(A.162)}\\
&= u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s})) - u'(\underline{s},\bar{e})\gamma\mathbb{P}(\underline{e}\mid\underline{s})\mathbb{P}(\underline{s},\bar{e}) - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) && \text{(A.163)}\\
&= u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})[1-\gamma\mathbb{P}(\bar{e}\mid\underline{s}) - \gamma\mathbb{P}(\underline{e}\mid\underline{s})] - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) && \text{(A.164)}\\
&= u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) && \text{(A.165)}\\
&= 0, && \text{(A.166)}
\end{aligned}$$

which is equivalent to $u'(\underline{s},\bar{e}) = u'(\bar{s},\bar{e})$ and, thus, $u'(\tilde{s},\tilde{e}) \equiv \xi$, i.e., buyers are fully insured. Thus, the optimal contract has the transfers from Proposition 8.

Defaults of fragile sellers For any margin $\alpha > \phi$, resources of fragile sellers are not sufficient after a negative signal at $t=1$, which induces them to default.

No defaults of safe seller Because $\alpha^* = \phi < 1$, the resource constraint (9) for safe sellers at $t=1$ is satisfied and, thus, safe sellers do not default at $t=1$.

The optimal transfers satisfy $T_S(\underline{s},\underline{e}) > T_S(\underline{s},\bar{e})$. Therefore, to satisfy the resource constraint

(10) in both states $(\underline{s},\underline{e})$ and $(\underline{s},\overline{e})$, it is necessary and sufficient that

$$
\begin{aligned}
& T_S(\underline{s},\underline{e}) \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}) && \text{(A.167)}\\
\Leftrightarrow & \pi + \gamma\frac{\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})}{1-\gamma} - m \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}) && \text{(A.168)}\\
\Leftrightarrow & (1-\gamma)\pi + \gamma(\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s}))\\
& - (1-\gamma)m \leq (1-\gamma)R(\underline{s}) - (1-\gamma)(1-k)\alpha R(\underline{s}) && \text{(A.169)}\\
\Leftrightarrow & \pi + \gamma(R(\underline{s})(1-(1-k)\alpha) - \underline{\pi} + C - \rho\phi R(\underline{s}))\\
& - m \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}) && \text{(A.170)}\\
\Leftrightarrow & \pi + \gamma(R(\underline{s})(1-(1-k)\alpha) - \underline{\pi} + C - \rho\phi R(\underline{s}))\\
& - \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\alpha R(\underline{s})) \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}) && \text{(A.171)}\\
\Leftrightarrow & \pi + \gamma[R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} + \mathbb{P}(\overline{s})C - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s})]\\
& \leq R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) && \text{(A.172)}\\
\Leftrightarrow & \gamma \leq \frac{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \pi}{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\overline{s})C} =: \overline{\gamma}^R, && \text{(A.173)}
\end{aligned}
$$

using that $m = \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\alpha R(\underline{s}))$. The numerator is strictly positive if and only if

$$
\begin{aligned}
& R(\underline{s}) > \mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) + \pi && \text{(A.174)}\\
\Leftrightarrow & \phi < \frac{R(\underline{s}) - \pi}{\mathbb{P}(\overline{s})(1-k)R(\underline{s})} =: \phi^*, && \text{(A.175)}
\end{aligned}
$$

which is the condition for the resource constraint being satisfied for $\gamma = 0$. Therefore, due to $R(\underline{s}) > \pi$ by assumption, the numerator is positive for all $\phi < \phi^*$ with $\phi^* > 0$.

The denominator of $\overline{\gamma}^R$ is strictly positive if and only if

$$
\begin{aligned}
& R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\overline{s})C > 0 && \text{(A.176)}\\
\Leftrightarrow\ & R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) - \underline{\pi} + \mathbb{P}(\overline{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) && \text{(A.177)}\\
\Leftrightarrow\ & R(\underline{s}) - \underline{\pi} + \mathbb{P}(\overline{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho) + \mathbb{P}(\overline{s})(1-k))\phi R(\underline{s}), && \text{(A.178)}
\end{aligned}
$$

using $\alpha = \phi$. Assuming that the numerator is strictly positive, a sufficient condition is that

$$
\begin{aligned}
& \mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) + \pi - \underline{\pi} + \mathbb{P}(\overline{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho) + \mathbb{P}(\overline{s})(1-k))\phi R(\underline{s}) && \text{(A.179)}\\
\Leftrightarrow\ & \pi - \underline{\pi} + \mathbb{P}(\overline{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) && \text{(A.180)}\\
\Leftrightarrow\ & \frac{\pi - \underline{\pi} + \mathbb{P}(\overline{s})C}{(\rho + \mathbb{P}(\underline{s})(1-\rho))R(\underline{s})} > \phi, && \text{(A.181)}
\end{aligned}
$$

which holds for all $C \geq 0$ if

$$\phi < \frac{\pi - \underline{\pi}}{(\rho + \mathbb{P}(\underline{s})(1-\rho))R(\underline{s})} =: \phi^{**}. \tag{A.182}$$

Therefore, a sufficient condition for both the numerator and denominator to be positive is that $\phi < \min\{\phi^*, \phi^{**}\} =: \phi^R$.

Moreover, it is

$$\begin{aligned}
&\overline{\gamma}^R < 1 && \text{(A.183)}\\
\Leftrightarrow\ & R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) - \pi < R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} \\
&\quad - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\bar{s})C && \text{(A.184)}\\
\Leftrightarrow\ & \pi - \underline{\pi} + \mathbb{P}(\bar{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}), && \text{(A.185)}
\end{aligned}$$

which is satisfied if $\phi < \phi^R$.

Finally, safe sellers do not default at $t = 1$ if and only if $\alpha \leq 1$, which holds because $\alpha^* = \phi < 1$.

Outsiders Outsiders have assets that pay R_O at $t = 2$ and make payments $T_S - p$. Thus, their resources are sufficient to make payments on the derivative contract if and only if

$$\begin{aligned}
& T_S(\underline{s}, \underline{e}) - p \leq R_O && \text{(A.186)}\\
\Leftrightarrow\ & T_S(\underline{s}, \underline{e}) - \mathbb{E}[T_S \mid \underline{s}] \leq R_O && \text{(A.187)}\\
\Leftrightarrow\ & \pi - m + \gamma(\mathbb{E}[T_S \mid \underline{s}] + C - \rho\phi R(\underline{s})) - \mathbb{E}[T_S \mid \underline{s}] \leq R_O && \text{(A.188)}\\
\Leftrightarrow\ & \pi - m + \gamma(C - \rho\phi R(\underline{s})) - (1-\gamma)\mathbb{E}[T_S \mid \underline{s}] \leq R_O && \text{(A.189)}\\
\Leftrightarrow\ & \pi - m + \gamma(C - \rho\phi R(\underline{s})) - (1-\gamma)\frac{\pi - \underline{\pi} - m + \gamma(C - \rho\phi R(\underline{s}))}{1-\gamma} \leq R_O && \text{(A.190)}\\
\Leftrightarrow\ & \pi - m + \gamma(C - \rho\phi R(\underline{s})) - (\pi - \underline{\pi} - m + \gamma(C - \rho\phi R(\underline{s}))) \leq R_O && \text{(A.191)}\\
\Leftrightarrow\ & \underline{\pi} \leq R_O. && \text{(A.192)}
\end{aligned}$$

Thus, using $\pi > \underline{\pi}$, it is sufficient that $R_O \geq \pi$. □

Proof of Proposition 10.

(1) Proposition 5 implies that the optimal margin under full insurance without replacement is either $\alpha^{NR} = 0$ or $\alpha^{NR} = \phi$. Hence,

$$\overline{\gamma}^{NR}(\alpha^{NR}) \leq \max\left\{\overline{\gamma}^{NR}(0), \overline{\gamma}^{NR}(\phi)\right\}. \tag{A.193}$$

Consider first the limit $\phi = 0$ and $C = 0$. In this case, the two no-replacement thresholds coincide:

$$\overline{\gamma}^{NR}(0) = \overline{\gamma}^{NR}(\phi) = \frac{R(\underline{s}) - \pi}{R(\underline{s})}. \tag{A.194}$$

In contrast, Proposition 9 implies that the full-insurance threshold with replacement is

$$\overline{\gamma}^{R} = \frac{R(\underline{s}) - \pi}{R(\underline{s}) - \underline{\pi}}. \tag{A.195}$$

Because $\underline{\pi} > 0$, it follows that

$$\overline{\gamma}^{R} > \overline{\gamma}^{NR}(0) = \overline{\gamma}^{NR}(\phi). \tag{A.196}$$

The three thresholds are continuous in ϕ and C around $\phi = C = 0$. Therefore, there exist $\widehat{\phi} > 0$ and $\widehat{C} > 0$ such that, for all $\phi < \widehat{\phi}$ and $C < \widehat{C}$,

$$\overline{\gamma}^{R} > \max\left\{\overline{\gamma}^{NR}(0), \overline{\gamma}^{NR}(\phi)\right\} \geq \overline{\gamma}^{NR}(\alpha^{NR}). \tag{A.197}$$

Thus, counterparty replacement enables full insurance for a strictly larger mass of fragile sellers than the optimal contract without replacement.

(2) If $\gamma > \overline{\gamma}^{NR}(\alpha^{NR})$, then Proposition 6 implies that the optimal contract without replacement provides less than full insurance to buyers, with expected utility

$$\mathbb{E}[u(\tilde{e} - T_B)] = u(\mathbb{E}[\tilde{e}] - m^{NR} - g^{NR}), \tag{A.198}$$

where $m^{NR} = \mathbb{E}[T_B]$ is the markup of that contract and $g^{NR} > 0$ is the risk premium for the remaining consumption risk $\tilde{e} - T_B$.

If $\gamma \leq \overline{\gamma}^{R}$, then Proposition 9 implies that the optimal contract with replacement provides full insurance, with expected utility

$$\mathbb{E}[u(\tilde{e} - T_B)] = u(\mathbb{E}[\tilde{e}] - m^{R}), \tag{A.199}$$

where $m^{R} = \mathbb{E}[T_B]$ is the markup of that contract.

Therefore, replacement is efficient if and only if

$$u(\mathbb{E}[\tilde{e}] - m^{NR} - g^{NR}) \leq u(\mathbb{E}[\tilde{e}] - m^{R}) \tag{A.200}$$

$$\Leftrightarrow m^{R} - m^{NR} \leq g^{NR}. \tag{A.201}$$

□

Proof of Example 1. Consider the full-insurance contract with replacement when $\phi = 0$ and $C = 0$.

Then,

$$\overline{\gamma}^R = \frac{R(\underline{s}) - \pi}{R(\underline{s}) - \underline{\pi}}, \tag{A.202}$$

$$\text{and } \overline{\gamma}^{NR} = \frac{R(\underline{s}) - \pi}{R(\underline{s})}. \tag{A.203}$$

Note that $\overline{\gamma}^{NR} < \overline{\gamma}^R < 1$.

Because $\gamma \leq \overline{\gamma}^R$, the optimal contract with replacement provides full insurance. Denote by $m^R = \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\phi R(\underline{s}))$ the markup of this contract. It is equal to $m^R = 0$ for $\phi = 0$ and $C = 0$.

Because $\gamma > \overline{\gamma}^{NR}$, the optimal contract without replacement provides partial insurance. The markup of this contract is either equal to $m^{NR}_{\text{wo}} = 0$ in the case without defaults at $t = 2$ or equal to $m^{NR}_{\text{w}} = \mathbb{P}(\underline{s})(1-k)\alpha R \geq 0$ in the case of defaults at $t = 2$.

Thus, the contract with replacement implements strictly more risk sharing and has a (weakly) lower markup $m^R = 0 \leq \min\{m^{NR}_{\text{wo}}, m^{NR}_{\text{w}}\}$. Thus, it dominates the optimal contract without replacement. □

Proof of Proposition 11. Define

$$\underline{q}(\lambda) := \mathbb{P}(\underline{s}) = (1-\lambda)\pi + \lambda(1-\pi), \tag{A.204}$$

$$\overline{q}(\lambda) := \mathbb{P}(\overline{s}) = 1 - \underline{q}(\lambda). \tag{A.205}$$

Hence,

$$\underline{q}'(\lambda) = 1 - 2\pi, \qquad \overline{q}'(\lambda) = 2\pi - 1. \tag{A.206}$$

Moreover,

$$\underline{\pi} = \frac{(1-\lambda)\pi}{\underline{q}(\lambda)}, \tag{A.207}$$

so that

$$\frac{\partial \underline{\pi}}{\partial \lambda} = -\frac{\pi(1-\pi)}{\underline{q}(\lambda)^2} < 0. \tag{A.208}$$

We first consider the full-insurance threshold with replacement. At $\alpha = \phi$, Proposition 9 gives

$$\overline{\gamma}^R = \frac{N^R}{D^R}, \tag{A.209}$$

where, suppressing the argument of $R(\underline{s})$ for readability,

$$N^R := R - \overline{q}(1-k)\phi R - \pi, \tag{A.210}$$

$$D^R := R - \overline{q}(1-k)\phi R - \underline{\pi} - \left[\rho + \underline{q}(1-\rho)\right]\phi R + \overline{q}C. \tag{A.211}$$

Assumption 3 implies $0 < \overline{\gamma}^R < 1$ and therefore

$$0 < N^R < D^R. \tag{A.212}$$

Using (A.206)–(A.208),

$$\frac{\partial N^R}{\partial \lambda} = -(2\pi - 1)(1-k)\phi R, \tag{A.213}$$

$$\frac{\partial D^R}{\partial \lambda} = -\frac{\partial \underline{\pi}}{\partial \lambda} + (2\pi - 1)\left[(k-\rho)\phi R + C\right]. \tag{A.214}$$

Consequently,

$$\frac{\partial \overline{\gamma}^R}{\partial \lambda} = \frac{N^R \frac{\partial \underline{\pi}}{\partial \lambda} - (2\pi - 1)\left[(1-k)\phi R D^R + N^R\big((k-\rho)\phi R + C\big)\right]}{(D^R)^2}. \tag{A.215}$$

The expression in square brackets can be rewritten as

$$\begin{aligned}&(1-k)\phi R D^R + N^R\big((k-\rho)\phi R + C\big)\\&\quad = \phi R\left[(1-k)(D^R - N^R) + (1-\rho)N^R\right] + N^R C \geq 0.\end{aligned} \tag{A.216}$$

Hence, if $\pi \geq 1/2$, then $2\pi - 1 \geq 0$ and

$$\frac{\partial \overline{\gamma}^R}{\partial \lambda} < 0. \tag{A.217}$$

We next consider the full-insurance threshold without replacement. Proposition 5 implies that the optimal full-insurance margin is either $\alpha^{NR} = 0$ or $\alpha^{NR} = \phi$. We consider the two cases separately.

No margin: $\alpha^{NR} = 0$. Since

$$\mathbb{P}(\underline{s}, \underline{e}) = \lambda(1-\pi), \tag{A.218}$$

the full-insurance threshold can be rewritten as

$$\overline{\gamma}^{NR}(0) = \frac{R - \pi}{R\left[1 - \big(\rho + \lambda(1-\pi)(1-\rho)\big)\phi\right]}. \tag{A.219}$$

Differentiating yields

$$\frac{\partial \overline{\gamma}^{NR}(0)}{\partial \lambda} = \frac{(R-\pi)(1-\pi)(1-\rho)\phi}{R\left[1-\left(\rho+\lambda(1-\pi)(1-\rho)\right)\phi\right]^2} \geq 0. \tag{A.220}$$

Combining (A.217) and (A.220) gives

$$\frac{\partial}{\partial \lambda}\left(\overline{\gamma}^R - \overline{\gamma}^{NR}(0)\right) < 0. \tag{A.221}$$

Full margin: $\alpha^{NR} = \phi$. In this case, Proposition 5 gives

$$\overline{\gamma}^{NR}(\phi) = \frac{R-\pi-\overline{q}(1-k)\phi R}{(1-\phi)R} = \frac{N^R}{R(1-\phi)}. \tag{A.222}$$

Therefore,

$$\frac{\partial \overline{\gamma}^{NR}(\phi)}{\partial \lambda} = -\frac{(2\pi-1)(1-k)\phi}{1-\phi} \leq 0. \tag{A.223}$$

Thus, unlike in the case without margin, both full-insurance thresholds decrease with signal informativeness.

To compare their rates of change, define

$$H := R(1-\phi) - D^R \tag{A.224}$$
$$= \underline{\pi} - \overline{q}\left[(k-\rho)\phi R + C\right]. \tag{A.225}$$

By assumption: $\overline{\gamma}^{NR} < \overline{\gamma}^R$. Because both thresholds have the same positive numerator N^R, this is equivalent to $D^R < R(1-\phi)$ and hence

$$H > 0. \tag{A.226}$$

Moreover, $\alpha^{NR} = \phi$ implies that Condition (17) does not hold at the full-insurance threshold:

$$\overline{\gamma}^{NR}(\phi)\mathbb{P}(\underline{s},\underline{e})(1-\rho) \geq \mathbb{P}(\underline{s})(1-k). \tag{A.227}$$

Because $\overline{\gamma}^{NR}(\phi) < 1$ and $\mathbb{P}(\underline{s},\underline{e}) < \mathbb{P}(\underline{s})$, Equation (A.227) implies

$$k > \rho. \tag{A.228}$$

Hence,

$$(k-\rho)\phi R + C \geq 0. \tag{A.229}$$

Using (A.206) and (A.208), it follows that

$$\frac{\partial H}{\partial \lambda} = \frac{\partial \pi}{\partial \lambda} - (2\pi - 1)\left[(k-\rho)\phi R + C\right] < 0. \tag{A.230}$$

Using $D^R = R(1-\phi) - H$, the difference in full-insurance capacity can be written as

$$\overline{\gamma}^R - \overline{\gamma}^{NR}(\phi) = \frac{N^R H}{D^R R(1-\phi)}. \tag{A.231}$$

Since $R(1-\phi)$ does not depend on λ, differentiating yields

$$\begin{aligned}
&\frac{\partial}{\partial \lambda}\left(\overline{\gamma}^R - \overline{\gamma}^{NR}(\phi)\right) \\
&= \frac{\left(\frac{\partial N^R}{\partial \lambda} H + N^R \frac{\partial H}{\partial \lambda}\right) D^R R(1-\phi) - N^R H \frac{\partial D^R}{\partial \lambda} R(1-\phi)}{\left[D^R R(1-\phi)\right]^2} \\
&= \frac{\frac{\partial N^R}{\partial \lambda} H}{D^R R(1-\phi)} + \frac{N^R \frac{\partial H}{\partial \lambda}}{D^R R(1-\phi)} - \frac{N^R H \frac{\partial D^R}{\partial \lambda}}{(D^R)^2 R(1-\phi)}.
\end{aligned} \tag{A.232}$$

Because

$$H = R(1-\phi) - D^R \tag{A.233}$$

and $R(1-\phi)$ does not depend on λ, it is

$$\frac{\partial D^R}{\partial \lambda} = -\frac{\partial H}{\partial \lambda}. \tag{A.234}$$

Therefore, the last two terms in (A.232) can be written as

$$\begin{aligned}
&\frac{N^R \frac{\partial H}{\partial \lambda}}{D^R R(1-\phi)} + \frac{N^R H \frac{\partial H}{\partial \lambda}}{(D^R)^2 R(1-\phi)} \\
&= \frac{N^R \frac{\partial H}{\partial \lambda}(D^R + H)}{(D^R)^2 R(1-\phi)} \\
&= \frac{N^R \frac{\partial H}{\partial \lambda}}{(D^R)^2},
\end{aligned} \tag{A.235}$$

where the last equality uses $D^R + H = R(1-\phi)$. Hence,

$$\frac{\partial}{\partial \lambda}\left(\overline{\gamma}^R - \overline{\gamma}^{NR}(\phi)\right) = \frac{\frac{\partial N^R}{\partial \lambda} H}{D^R R(1-\phi)} + \frac{N^R \frac{\partial H}{\partial \lambda}}{(D^R)^2} < 0, \tag{A.236}$$

where the inequality follows from $\partial N^R / \partial \lambda \leq 0$, $H > 0$, $N^R > 0$, and $\partial H / \partial \lambda < 0$.

□

A.3.2 Partial Insurance

Proof of Proposition 12. The optimal contract with replacement maximizes expected buyer utility subject to the CCP's budget constraints, the sellers' participation constraint, the resource constraints of safe sellers, and the requirement that fragile sellers default at $t = 1$. Let η denote the multiplier on the CCP's budget constraint, ξ that on the sellers' participation constraint, $\sigma_{S,2}$ that on the resource constraint of safe sellers at $t = 2$, and $\sigma_{F,1}$ that on the constraint inducing fragile sellers to default at $t = 1$.

By assumption,

$$\sigma_{S,2}(\underline{s}, \underline{e}) > 0, \qquad \sigma_{S,2}(\underline{s}, \bar{e}) = 0, \tag{A.237}$$

and hence

$$T_S(\underline{s}, \underline{e}) = \big(1 - (1-k)\alpha\big) R(\underline{s}). \tag{A.238}$$

Risk sharing. The first-order condition with respect to buyer transfers is

$$\eta(\tilde{s}, \tilde{e}) = \mathbb{P}(\tilde{s}, \tilde{e}) u'(\tilde{s}, \tilde{e}). \tag{A.239}$$

Conditional on a good signal, the first-order condition with respect to seller transfers is

$$\eta(\bar{s}, \tilde{e}) = \xi \mathbb{P}(\bar{s}, \tilde{e}), \tag{A.240}$$

which together with (A.239) implies

$$\xi = u'(\bar{s}, \underline{e}) = u'(\bar{s}, \bar{e}). \tag{A.241}$$

Thus, buyers are fully insured conditional on the good signal.

After a bad signal, the first-order conditions with respect to seller transfers are

$$0 = \eta(\underline{s}, \underline{e})\big(1 - \gamma(1 - \underline{\pi})\big) - \eta(\underline{s}, \bar{e})\gamma(1 - \underline{\pi}) - \xi \mathbb{P}(\underline{s}, \underline{e})(1 - \gamma) - \sigma_{S,2}(\underline{s}, \underline{e}), \tag{A.242}$$

$$0 = \eta(\underline{s}, \bar{e})\big(1 - \gamma\underline{\pi}\big) - \eta(\underline{s}, \underline{e})\gamma\underline{\pi} - \xi \mathbb{P}(\underline{s}, \bar{e})(1 - \gamma) - \sigma_{S,2}(\underline{s}, \bar{e}). \tag{A.243}$$

Dividing (A.242) and (A.243) by the corresponding state probabilities, subtracting, and using (A.239) gives

$$u'(\underline{s}, \underline{e}) - u'(\underline{s}, \bar{e}) = \frac{\sigma_{S,2}(\underline{s}, \underline{e})}{\mathbb{P}(\underline{s}, \underline{e})} - \frac{\sigma_{S,2}(\underline{s}, \bar{e})}{\mathbb{P}(\underline{s}, \bar{e})}. \tag{A.244}$$

Using (A.237),

$$u'(\underline{s},\underline{e}) > u'(\underline{s},\bar{e}). \tag{A.245}$$

Next, setting $\sigma_{S,2}(\underline{s},\bar{e}) = 0$ in (A.243), and using (A.239) and (A.241), gives

$$u'(\underline{s},\bar{e})(1-\gamma\underline{\pi}) = \gamma(1-\underline{\pi})u'(\underline{s},\underline{e}) + (1-\gamma)u'(\bar{s},\bar{e}), \tag{A.246}$$

or equivalently,

$$u'(\underline{s},\bar{e}) = \frac{\gamma(1-\underline{\pi})}{1-\gamma\underline{\pi}}u'(\underline{s},\underline{e}) + \frac{1-\gamma}{1-\gamma\underline{\pi}}u'(\bar{s},\bar{e}). \tag{A.247}$$

The two coefficients on the right-hand side are strictly positive and sum to one. Together with (A.245), this implies

$$u'(\underline{s},\underline{e}) > u'(\underline{s},\bar{e}) > u'(\bar{s},\bar{e}) = u'(\bar{s},\underline{e}). \tag{A.248}$$

By concavity of $u(\cdot)$, buyer utility therefore satisfies

$$u(\underline{s},\underline{e}) < u(\underline{s},\bar{e}) < u(\bar{s},\underline{e}) = u(\bar{s},\bar{e}). \tag{A.249}$$

Markup. Taking expectations of the CCP's budget constraints and using $p = \mathbb{E}[T_S \mid \underline{s}]$ gives

$$\mathbb{E}[T_B] - \mathbb{P}(\underline{s})\gamma C + \mathbb{P}(\underline{s})\gamma\rho\phi R(\underline{s}) = -\mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\mathbb{E}[T_S \mid \underline{s}]. \tag{A.250}$$

The binding participation constraint of protection sellers is

$$0 = -\mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\big(\mathbb{E}[T_S \mid \underline{s}] - \phi R(\underline{s})\big) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s}). \tag{A.251}$$

Combining (A.250) and (A.251) yields

$$\mathbb{E}[T_B] = \mathbb{P}(\underline{s})\Big(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\alpha R(\underline{s})\Big). \tag{A.252}$$

Optimal margin. The first-order condition with respect to the margin requirement is

$$0 = -\xi\mathbb{P}(\underline{s})(1-\gamma)(1-k)R(\underline{s}) + \sigma_{F,1} - \sigma_{S,2}(\underline{s},\underline{e})(1-k)R(\underline{s}), \tag{A.253}$$

where we have used $\sigma_{S,2}(\underline{s},\bar{e}) = 0$. Hence,

$$\sigma_{F,1} = \Big[\xi\mathbb{P}(\underline{s})(1-\gamma) + \sigma_{S,2}(\underline{s},\underline{e})\Big](1-k)R(\underline{s}) > 0. \tag{A.254}$$

The constraint that induces fragile sellers to default therefore binds. Thus, using the convention

$$\alpha^* = \inf\{\alpha : \alpha > \phi\}, \tag{A.255}$$

the optimal margin is

$$\alpha = \alpha^* = \phi. \tag{A.256}$$

Substituting this into (A.252) gives

$$m = \mathbb{P}(\underline{s})\Big(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\phi R(\underline{s})\Big), \tag{A.257}$$

and the binding resource constraint of safe sellers becomes

$$T_S(\underline{s}, \underline{e}) = \big(1 - (1-k)\phi\big) R(\underline{s}). \tag{A.258}$$

$\square$

Proof of Proposition 13. We first characterize the restricted replacement benchmark. As in the unrestricted replacement contract, fragile sellers must default at $t = 1$. Because posting margin is costly, the optimal margin is therefore the smallest margin that induces their default,

$$\alpha^R = \phi. \tag{A.259}$$

The markup is consequently the same as under the unrestricted replacement contract,

$$m^R = \mathbb{P}(\underline{s})\Big(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\phi R(\underline{s})\Big). \tag{A.260}$$

Because $\gamma > \overline{\gamma}^R$, full insurance is not resource compatible. We define the restricted class of contracts as those that equalize buyer consumption across all states except $(\underline{s}, \underline{e})$. Optimal risk sharing requires the resource constraint of safe sellers in the worst state to bind:

$$T_S^{R,res}(\underline{s}, \underline{e}) = \big[1 - (1-k)\phi\big] R(\underline{s}). \tag{A.261}$$

The restriction (41) implies

$$T_B^{R,res}(\overline{s}, \overline{e}) = T_B^{R,res}(\underline{s}, \overline{e}), \tag{A.262}$$

$$T_B^{R,res}(\overline{s}, \underline{e}) = T_B^{R,res}(\underline{s}, \overline{e}) - 1. \tag{A.263}$$

After a negative signal, the CCP budget constraints imply

$$T_B^{R,res}(\underline{s},\bar{e}) = -T_S^{R,res}(\underline{s},\bar{e}) - \gamma\rho\phi R(\underline{s}) + \gamma\Big(\mathbb{E}[T_S^{R,res} \mid \underline{s}] + C\Big), \tag{A.264}$$

$$T_B^{R,res}(\underline{s},\underline{e}) = -\big[1-(1-k)\phi\big]R(\underline{s}) - \gamma\rho\phi R(\underline{s}) + \gamma\Big(\mathbb{E}[T_S^{R,res} \mid \underline{s}] + C\Big). \tag{A.265}$$

Moreover,

$$\mathbb{E}[T_S^{R,res} \mid \underline{s}] = (1-\underline{\pi})\big[1-(1-k)\phi\big]R(\underline{s}) + \underline{\pi}T_S^{R,res}(\underline{s},\bar{e}). \tag{A.266}$$

Given the restriction on risk sharing and the binding resource constraint (A.261), the remaining transfer after a negative signal can be parameterized by $T_S^{R,res}(\underline{s},\bar{e})$. The restricted benchmark chooses this transfer to maximize buyer utility subject to the CCP budget constraints and sellers' participation constraint. The latter binds and, together with the CCP budget constraints, implies $\mathbb{E}[T_B^{R,res}] = m^R$.

To see how this transfer determines risk sharing, subtracting (A.265) from (A.264) gives

$$T_B^{R,res}(\underline{s},\bar{e}) - T_B^{R,res}(\underline{s},\underline{e}) = \big[1-(1-k)\phi\big]R(\underline{s}) - T_S^{R,res}(\underline{s},\bar{e}). \tag{A.267}$$

Hence, the difference between buyer consumption in the non-worst and worst states is

$$\Delta^{R,res} = 1 - \big[1-(1-k)\phi\big]R(\underline{s}) + T_S^{R,res}(\underline{s},\bar{e}). \tag{A.268}$$

Using (A.262)–(A.266), (A.268), and $\mathbb{E}[T_B^{R,res}] = m^R$ then pins down the remaining transfer and yields

$$\Delta^{R,res} = \frac{\pi - \gamma\underline{\pi} - m^R - (1-\gamma)\big[1-(1-k)\phi\big]R(\underline{s}) - \gamma\big[\rho\phi R(\underline{s}) - C\big]}{1 - \mathbb{P}(\underline{s},\underline{e}) - \gamma\underline{\pi}}. \tag{A.269}$$

We next consider the contract without replacement. By assumption, the optimal contract without replacement involves defaults of fragile sellers at $t=2$. Safe sellers' resource constraint binds in $(\underline{s},\underline{e})$, such that their maximum transfer is

$$\big[1-(1-k)\alpha^{NR}\big]R(\underline{s}). \tag{A.270}$$

Fragile sellers that default in this state contribute

$$\big[\alpha^{NR}k + \rho(\phi - \alpha^{NR})\big]R(\underline{s}) \tag{A.271}$$

to the CCP. Because buyer consumption is equalized across all remaining states, the corresponding consumption gap is

$$\Delta^{NR} = \frac{\pi - m^{NR} - (1-\gamma)\left[1-(1-k)\alpha^{NR}\right]R(\underline{s}) - \gamma\left[\alpha^{NR}k + \rho(\phi - \alpha^{NR})\right]R(\underline{s})}{1-\mathbb{P}(\underline{s},\underline{e})}. \tag{A.272}$$

It remains to compare expected utility. Both contracts generate two consumption levels, where the low level occurs only in state $(\underline{s},\underline{e})$. Let c_+^j and c_-^j denote these two levels for $j \in \{R^{res}; NR\}$. Since $\mathbb{E}[\tilde{e} - T_B] = \pi - m^j$, they satisfy

$$c_+^j = \pi - m^j + \mathbb{P}(\underline{s},\underline{e})\Delta^j, \tag{A.273}$$

$$c_-^j = \pi - m^j - \left[1-\mathbb{P}(\underline{s},\underline{e})\right]\Delta^j. \tag{A.274}$$

Suppose first that $m^R = m^{NR}$. If $\Delta^{R,res} < \Delta^{NR}$, the restricted replacement allocation is a mean-preserving contraction of the allocation without replacement. Concavity of $u(\cdot)$ therefore implies strictly greater expected buyer utility under the restricted replacement benchmark.

More generally, if $m^R \leq m^{NR}$, increasing consumption in both states of the no-replacement allocation by $m^{NR} - m^R$ weakly raises buyer utility and gives the same mean consumption as the restricted replacement benchmark. If, in addition, $\Delta^{R,res} \leq \Delta^{NR}$, the restricted replacement allocation is a weak mean-preserving contraction of this shifted allocation. Hence, expected buyer utility is weakly larger under the restricted replacement benchmark, and strictly larger if at least one of the two inequalities is strict.

Finally, the unrestricted optimal contract with replacement maximizes buyer utility over a larger set of contracts than the restricted benchmark. It therefore provides weakly greater expected utility than the restricted replacement benchmark. Conditions (42) and (43) consequently imply that the optimal contract with replacement strictly dominates the optimal contract without replacement.

Polar case. Let $\phi = C = 0$. Feasibility implies

$$\alpha^{NR} = 0, \qquad \alpha^R = 0, \tag{A.275}$$

and therefore

$$m^R = m^{NR} = 0. \tag{A.276}$$

Equations (A.269) and (A.272) reduce to

$$\Delta^{R,res} = \frac{\pi - \gamma\underline{\pi} - (1-\gamma)R(\underline{s})}{1-\mathbb{P}(\underline{s},\underline{e}) - \gamma\underline{\pi}}, \tag{A.277}$$

$$\Delta^{NR} = \frac{\pi - (1-\gamma)R(\underline{s})}{1-\mathbb{P}(\underline{s},\underline{e})}. \tag{A.278}$$

It follows that

$$\Delta^{R,res} < \Delta^{NR} \tag{A.279}$$

if and only if

$$\gamma\underline{\pi}\Big(\mathbb{P}(\bar{s},\underline{e}) + (1-\gamma)R(\underline{s})\Big) > 0. \tag{A.280}$$

This inequality holds strictly for $\gamma > 0$ and $\underline{\pi} > 0$. Hence, the restricted replacement benchmark provides strictly more insurance at the same markup. It therefore gives buyers strictly greater expected utility than the optimal contract without replacement. The unrestricted optimal contract with replacement dominates the restricted benchmark and thus strictly dominates the optimal contract without replacement. □

B Replacement by Incumbent Sellers

This appendix considers an extension of the model in which the CCP does not rely on outside sellers to replace defaulted counterparties. Instead, all protection sellers are present at $t = 0$ and are ex-ante identical. After a negative signal at $t = 1$, sellers privately learn whether they are fragile, mediocre, or safe. The CCP can then use margin calls to identify fragile sellers and direct their positions to safe sellers from the original clearing-member pool.

B.1 Summary of results

The extension delivers three results. First, allowing for an intermediate seller type generates the possibility of cascading defaults within the seller pool. If fragile sellers default at $t = 2$, the resulting loss-sharing contribution may exceed the resources of mediocre sellers, causing them to default as well. The remaining losses must then be borne by safe sellers, which can make full insurance infeasible.

Second, replacement by incumbent sellers can prevent this cascade. The CCP uses a first margin call α to induce fragile sellers to default. It then assigns the defaulted positions to sellers that satisfy a second margin requirement. The two margin requirements screen sellers according to their balance-sheet resources: mediocre sellers can maintain their original positions but cannot take over defaulted positions, whereas safe sellers can do both. As in the main model, the cost-minimizing margins are the smallest margins that implement the required separation.

Third, replacement by incumbent sellers can strictly increase welfare through two distinct channels. In a tractable benchmark with fully impaired fragile sellers and no deadweight replacement cost, the optimal no-replacement contract either lets only fragile sellers or both, fragile and mediocre, sellers default at $t = 2$. In the first case, replacement is efficient when buyers' willingness to pay for the additional insurance exceeds the opportunity cost of the canary margins. In the second case, replacement additionally avoids the deadweight loss from mediocre-seller defaults; we provide a condition under which replacement then strictly dominates the optimal no-replacement contract.

B.2 Three seller types

The timing, endowment risk, public signal, transfers, and CCP budget constraints are as in the main model. We modify only the distribution of seller resources following a negative signal. Conditional on $\tilde{s} = \underline{s}$, each seller privately learns its type $j \in \{F, M, S\}$. The masses of fragile, mediocre, and safe sellers are denoted by γ_F, γ_M, and γ_S, respectively, where

$$\gamma_F + \gamma_M + \gamma_S = 1. \tag{A.281}$$

A type-j seller has assets that pay $\phi_j R(\underline{s})$ at $t = 2$, with

$$0 \leq \phi_F < \phi_M < \phi_S = 1. \tag{A.282}$$

Thus, sellers are identical at $t = 0$ and differ only in their privately observed balance-sheet resources at $t = 1$.

We retain the replacement cost $C \geq 0$ from the main model. Here, C captures the deadweight cost of transferring a defaulted position to another clearing member. In contrast to the baseline model with outsiders, there is no replacement payment p. We assume that the contingent obligation to take over defaulted positions is part of the clearing arrangement agreed at $t = 0$.

B.3 Full insurance without replacement

We first characterize the default cascade that can arise without replacement. As in the corresponding full-insurance benchmark in the main model, we consider contracts in which fragile sellers survive the interim date and default at $t = 2$ in state $(\underline{s}, \underline{e})$.

Suppose fragile sellers default in state $(\underline{s}, \underline{e})$. With no margins, the markup is

$$m_F^{NR} = \mathbb{P}(\underline{s}, \underline{e})(1-\rho)\gamma_F \phi_F R(\underline{s}). \tag{A.283}$$

The transfer required from each surviving seller in the default state is then

$$T_F^{NR} := T_S(\underline{s}, \underline{e}) = \frac{\pi - [\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho)]\, \gamma_F \phi_F R(\underline{s})}{1-\gamma_F}. \tag{A.284}$$

Mediocre sellers default if $T_F^{NR} > \phi_M R(\underline{s})$. If they default together with fragile sellers, the markup becomes

$$m_{FM}^{NR} = \mathbb{P}(\underline{s}, \underline{e})(1-\rho)(\gamma_F \phi_F + \gamma_M \phi_M) R(\underline{s}), \tag{A.285}$$

and the transfer required from each remaining safe seller is

$$T_{FM}^{NR} := T_S(\underline{s}, \underline{e}) = \frac{\pi - [\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho)]\, (\gamma_F \phi_F + \gamma_M \phi_M) R(\underline{s})}{\gamma_S}. \tag{A.286}$$

Proposition A.1 (Default cascade without replacement)**.** *Suppose fragile sellers cannot make the first-best transfer,*

$$\phi_F R(\underline{s}) < \pi. \tag{A.287}$$

If

$$\pi > [(\gamma_M + \gamma_S)\phi_M + (\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho))\, \gamma_F \phi_F]\, R(\underline{s}), \tag{A.288}$$

$$\textit{and } \pi > [\gamma_S + (\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho))\, (\gamma_F \phi_F + \gamma_M \phi_M)]\, R(\underline{s}), \tag{A.289}$$

then the default of fragile sellers triggers the default of mediocre sellers and full insurance is not resource-compatible without replacement.

The first condition in Proposition A.1 implies that the transfer required from surviving sellers after fragile sellers default exceeds mediocre sellers' resources. The second condition implies that, after mediocre sellers default as well, safe sellers cannot absorb the remaining losses. Hence the CCP cannot maintain full insurance through ex-post loss sharing alone.

B.4 Full insurance with replacement by incumbent sellers

We now allow the CCP to replace fragile sellers at $t = 1$. The CCP first calls margin α on each original position. To induce fragile sellers to default while mediocre and safe sellers survive, the margin satisfies

$$\phi_F < \alpha \le \phi_M. \tag{A.290}$$

After fragile sellers default, the CCP allocates their positions to safe sellers. We assume that replacement positions are assigned in fixed, indivisible bundles, with each replacing seller receiving

$$\ell := \frac{\gamma_F}{\gamma_S} \tag{A.291}$$

defaulted positions. Each replacement position carries an additional margin β. A seller that accepts a replacement bundle must therefore be able to post total margin $\alpha + \ell\beta$. To exclude mediocre sellers from replacement while allowing safe sellers to take the bundle, margins must satisfy

$$\phi_M < \alpha + \ell\beta \le 1. \tag{A.292}$$

Because the mass $1 - \gamma_F$ of surviving original sellers posts α and the mass γ_F of replacement positions posts β, aggregate margin is

$$(1 - \gamma_F)\alpha + \gamma_F\beta. \tag{A.293}$$

For any given α, the smallest replacement margin that separates mediocre and safe sellers is $\beta = (\phi_M - \alpha)/\ell$ in the limiting sense. Substituting this margin into Equation (A.293) gives

$$\gamma_M\alpha + \gamma_S\phi_M, \tag{A.294}$$

which is strictly increasing in α. Hence the cost-minimizing screening margins are

$$\alpha^* = \phi_F, \qquad \beta^* = \frac{\phi_M - \phi_F}{\ell}. \tag{A.295}$$

As before, these expressions denote limiting values: implemented margins are arbitrarily above the strict screening thresholds.

Following replacement, the total mass of active positions remains one. The CCP budget constraint after a negative signal is therefore

$$T_B(\underline{s}, \tilde{e}) + T_S(\underline{s}, \tilde{e}) + \gamma_F \rho \phi_F R(\underline{s}) - \gamma_F C = 0. \tag{A.296}$$

The transfers avoided by fragile sellers after their default are exactly offset by the transfers on the same positions made by safe replacement sellers. Protection sellers' ex-ante participation constraint is consequently

$$\begin{aligned} 0 = &-\mathbb{E}[T_S] - \mathbb{P}(\underline{s})\gamma_F \phi_F R(\underline{s}) \\ &- \mathbb{P}(\underline{s})(1-k)\left[(1-\gamma_F)\alpha + \gamma_F \beta\right] R(\underline{s}). \end{aligned} \tag{A.297}$$

Combining Equations (A.296) and (A.297) gives the markup at the cost-minimizing margins,

$$m^R = \mathbb{P}(\underline{s})\Big[\gamma_F C + \gamma_F(1-\rho)\phi_F R(\underline{s}) + (1-k)(\gamma_M \phi_F + \gamma_S \phi_M) R(\underline{s})\Big]. \tag{A.298}$$

Under full insurance, transfers from sellers are

$$T_S(\bar{s}, \bar{e}) = -(1-\pi) - m^R, \tag{A.299}$$

$$T_S(\bar{s}, \underline{e}) = \pi - m^R, \tag{A.300}$$

$$T_S(\underline{s}, \bar{e}) = -(1-\pi) - m^R + \gamma_F\left[C - \rho \phi_F R(\underline{s})\right], \tag{A.301}$$

$$T_S(\underline{s}, \underline{e}) = \pi - m^R + \gamma_F\left[C - \rho \phi_F R(\underline{s})\right]. \tag{A.302}$$

At $t = 2$, a mediocre seller has resources

$$\bar{t}_{M,2} = \left[\phi_M - (1-k)\phi_F\right] R(\underline{s}), \tag{A.303}$$

whereas a safe seller posts total margin $\alpha^* + \ell\beta^* = \phi_M$ and therefore has resources

$$\bar{t}_{S,2} = \left[1 - (1-k)\phi_M\right] R(\underline{s}). \tag{A.304}$$

A safe seller carries its original position and ℓ replacement positions, so its total payment obligation is $(1+\ell)T_S$.

Proposition A.2 (Full insurance with replacement by incumbent sellers)**.** *Suppose $0 \leq \phi_F < \phi_M < 1$ and replacement positions are allocated according to the rule in Equation (A.291). The cost-minimizing canary margins are given by Equation (A.295). The full-insurance contract* *has markup given by Equation (A.298) and transfers given by Equations (A.299)–(A.302). It is resource-compatible if and only if*

$$T_S(\underline{s}, \underline{e}) \leq \bar{t}_{M,2} \tag{A.305}$$

$$\textit{and } (1+\ell)T_S(\underline{s}, \underline{e}) \leq \bar{t}_{S,2}. \tag{A.306}$$

Proposition A.2 shows that the two surviving types perform different roles. Mediocre sellers maintain their original positions, whereas safe sellers provide the additional risk-sharing capacity required to replace fragile sellers.

B.5 When is incumbent replacement efficient?

To obtain a transparent comparison, consider the benchmark in which fragile sellers are completely impaired and transferring positions does not generate a deadweight replacement cost,

$$\phi_F = 0, \qquad C = 0. \tag{A.307}$$

Set mediocre sellers' resources to the lowest level at which they can make the first-best transfer on their original position,

$$\phi_M = \frac{\pi}{R(\underline{s})}, \tag{A.308}$$

and parameterize the composition of the surviving seller pool as

$$\gamma_M = (1-\omega)(1-\gamma_F), \qquad \gamma_S = \omega(1-\gamma_F), \qquad \omega \in (0,1). \tag{A.309}$$

Without replacement, fragile sellers default at $t = 2$. Under a full-insurance contract with late defaults, the transfer required from each surviving seller after the fragile sellers' default is $\pi/(1-\gamma_F) > \pi = \phi_M R(\underline{s})$ and, thus, mediocre sellers default as well. Full insurance in this benchmark is therefore feasible if and only if

$$R(\underline{s}) \geq \frac{\pi - (1-\gamma_F)(1-\omega)\left[\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho)\right]\pi}{(1-\gamma_F)\omega}, \tag{A.310}$$

which is equivalent to

$$\gamma_F \leq \overline{\gamma}_F^{NR}, \tag{A.311}$$

where

$$\overline{\gamma}_F^{NR} := 1 - \frac{\pi}{\omega R(\underline{s}) + [\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho)]\,(1-\omega)\pi}. \tag{A.312}$$

Optimal contract without replacement. We next characterize the optimal no-replacement contract in the region in which full insurance is infeasible, $\gamma_F > \overline{\gamma}_F^{NR}$. As in the no-replacement benchmark in the main text, we consider contracts in which sellers survive the interim date and defaults occur at $t = 2$. Because $\phi_F = 0$, this requires $\alpha = 0$.

We first rule out a third candidate: a contract without defaults. If fragile sellers do not default, their resource constraint binds in the worst state. Since $\phi_F = 0$, the optimal no-default contract has no margin and

$$T_S(\underline{s}, \underline{e}) = 0. \tag{A.313}$$

The contract is actuarially fair. Buyer consumption is therefore zero in the worst state and, by optimal risk sharing across the remaining states, equal to

$$\frac{\pi}{1 - \mathbb{P}(\underline{s}, \underline{e})} \tag{A.314}$$

in each other state. Allowing fragile sellers to default relaxes their resource constraint without generating a deadweight loss because their resources are zero. As shown below, the contract in which only fragile sellers default is also actuarially fair but raises worst-state consumption to $(1-\gamma_F)\pi > 0$. It is therefore a strict mean-preserving contraction of the no-default allocation. Thus, the no-default contract cannot be optimal.

There are consequently only two relevant partial-insurance contracts. In the first, only fragile sellers default in state $(\underline{s}, \underline{e})$. Since fragile sellers have no resources, their default generates no deadweight loss and the contract is actuarially fair. Mediocre sellers are the constrained surviving type, so their resource constraint binds,

$$T_S(\underline{s}, \underline{e}) = \phi_M R(\underline{s}) = \pi. \tag{A.315}$$

Buyer consumption in the worst state and in each of the remaining states is therefore, respectively,

$$c_F^{NR} := (1-\gamma_F)\pi, \tag{A.316}$$

$$\bar{c}_F^{NR} := \frac{\pi - \mathbb{P}(\underline{s}, \underline{e})(1-\gamma_F)\pi}{1 - \mathbb{P}(\underline{s}, \underline{e})}. \tag{A.317}$$

In the second contract, fragile and mediocre sellers default in state $(\underline{s}, \underline{e})$. The markup equals the deadweight loss from mediocre-seller defaults,

$$m_{FM}^{NR} = \mathbb{P}(\underline{s}, \underline{e})(1-\rho)\gamma_M \pi. \tag{A.318}$$

Because $\gamma_F > \overline{\gamma}_F^{NR}$, safe sellers cannot implement full insurance. Their resource constraint therefore binds, such that $T_S(\underline{s}, \underline{e}) = R(\underline{s})$. Buyer consumption is

$$c_{FM}^{NR} := \gamma_S R(\underline{s}) + \rho\gamma_M \pi, \tag{A.319}$$

$$\bar{c}_{FM}^{NR} := \frac{\pi - \mathbb{P}(\underline{s}, \underline{e})\left[\gamma_M \pi + \gamma_S R(\underline{s})\right]}{1 - \mathbb{P}(\underline{s}, \underline{e})}. \tag{A.320}$$

The first contract dominates if allowing mediocre sellers to default does not raise consumption in the worst state. In particular,

$$c_{FM}^{NR} - c_F^{NR} = \gamma_S \left[R(\underline{s}) - \pi\right] - (1-\rho)\gamma_M \pi. \tag{A.321}$$

Thus, a necessary condition for defaults of mediocre sellers to be optimal is

$$\gamma_S \left[R(\underline{s}) - \pi\right] > (1-\rho)\gamma_M \pi. \tag{A.322}$$

When this condition holds, allowing mediocre sellers to default raises consumption in the worst state but lowers consumption in all other states. The optimal default regime therefore depends on buyers' risk aversion.

Proposition A.3 (Optimal contract without replacement)**.** *Suppose Equations (A.307)–(A.309) hold and $\gamma_F > \overline{\gamma}_F^{NR}$. The optimal no-replacement contract uses no margins and provides partial insurance. A contract without defaults is strictly dominated by a contract in which only fragile sellers default. Hence the optimal contract is one of the following two contracts:*

(A) only fragile sellers default at $t = 2$, with buyer consumption given by Equations (A.316)–(A.317);

(B) fragile and mediocre sellers default at $t = 2$, with buyer consumption given by Equations (A.319)–(A.320).

The second contract is optimal if and only if

$$\begin{aligned} &\mathbb{P}(\underline{s}, \underline{e}) u(c_{FM}^{NR}) + \left[1 - \mathbb{P}(\underline{s}, \underline{e})\right] u(\bar{c}_{FM}^{NR}) \\ &\qquad \geq \mathbb{P}(\underline{s}, \underline{e}) u(c_F^{NR}) + \left[1 - \mathbb{P}(\underline{s}, \underline{e})\right] u(\bar{c}_F^{NR}). \end{aligned} \tag{A.323}$$

If Condition (A.322) fails, only fragile sellers default in the optimal contract.

Condition (A.323) shows what is required for the default cascade to be optimal rather feasible. Default by mediocre sellers releases the additional balance-sheet capacity of safe sellers, increasing worst-state consumption by Equation (A.321), but destroys the fraction $1-\rho$ of mediocre sellers' assets. Hence mediocre sellers default optimally only if buyers value the resulting additional insurance sufficiently strongly. Under the parametrization in Equation (A.309), the necessary condition becomes

$$\frac{\omega}{1-\omega}\left[R(\underline{s})-\pi\right] > (1-\rho)\pi. \tag{A.324}$$

A convenient sufficient condition for Equation (A.323) is

$$\frac{u'(c_{FM}^{NR})}{u'(\bar{c}_{FM}^{NR})} \geq \frac{\gamma_S[R(\underline{s})-\pi]}{\gamma_S[R(\underline{s})-\pi]-(1-\rho)\gamma_M\pi}, \tag{A.325}$$

provided Condition (A.322) holds. This condition requires sufficiently strong curvature of buyer utility: the marginal value of resources in the worst state must be sufficiently large relative to that in the remaining states.

With replacement, the canary margins satisfy $\alpha^* = 0$ and $\ell\beta^* = \phi_M$. The replacement markup becomes

$$m^R = \mathbb{P}(\underline{s})(1-k)\omega(1-\gamma_F)\pi. \tag{A.326}$$

Mediocre sellers can make their one-position payment because $T_S(\underline{s},\underline{e}) = \pi - m^R \leq \pi$. A convenient sufficient condition for the safe-seller resource constraint is

$$\left(1+\frac{\gamma_F}{\gamma_S}\right)\pi \leq R(\underline{s})-(1-k)\pi, \tag{A.327}$$

where we conservatively ignore the reduction in the transfer generated by the markup m^R. Using Equation (A.309), this condition is equivalent to

$$\gamma_F \leq \frac{\omega\left[R(\underline{s})-(2-k)\pi\right]}{\pi(1-\omega)+\omega\left[R(\underline{s})-(1-k)\pi\right]}. \tag{A.328}$$

As in the main text, greater risk sharing does not by itself imply that replacement is efficient because screening margins generate a markup. The welfare comparison depends on which of the two contracts in Proposition A.3 is optimal without replacement. We therefore distinguish the two cases explicitly.

Case 1: only fragile sellers default without replacement. Suppose Condition (A.323) does not hold, such that it is optimal without replacement to let only fragile sellers default. Because $\phi_F = 0$, these defaults generate no deadweight loss and the no-replacement contract is actuarially fair. Define its consumption risk premium $g_F^{NR} > 0$ by

$$\mathbb{P}(\underline{s}, \underline{e}) u(c_F^{NR}) + [1 - \mathbb{P}(\underline{s}, \underline{e})] \, u(\bar{c}_F^{NR}) = u\left(\pi - g_F^{NR}\right). \tag{A.329}$$

Replacement provides full insurance at constant consumption $\pi - m^R$. It is therefore efficient relative to the optimal no-replacement contract if and only if

$$m^R \leq g_F^{NR}. \tag{A.330}$$

This case highlights a pure risk-sharing benefit of replacement. Replacement can be optimal even though it would not be optimal to let mediocre sellers default without replacement. In particular, if Condition (A.322) fails, the contract with only fragile-seller defaults strictly dominates the contract with joint fragile- and mediocre-seller defaults. Nevertheless, replacement strictly dominates both contracts whenever $m^R < g_F^{NR}$.

Case 2: fragile and mediocre sellers default without replacement. Suppose instead that Condition (A.323) holds, such that it is optimal without replacement to let both fragile and mediocre sellers default. Define the consumption risk premium $g_{FM}^{NR} > 0$ of this contract by

$$\mathbb{P}(\underline{s}, \underline{e}) u(c_{FM}^{NR}) + [1 - \mathbb{P}(\underline{s}, \underline{e})] \, u(\bar{c}_{FM}^{NR}) = u\left(\pi - m_{FM}^{NR} - g_{FM}^{NR}\right). \tag{A.331}$$

Replacement is then efficient if and only if

$$m^R - m_{FM}^{NR} \leq g_{FM}^{NR}. \tag{A.332}$$

In this case, replacement not only increases risk sharing but also prevents the default of mediocre sellers. This yields a stronger sufficient condition: replacement is strictly efficient whenever its markup is weakly smaller than the markup generated by mediocre-seller defaults.

Proposition A.4 (Efficiency of replacement by incumbent sellers)**.** *Suppose Equations (A.307)–*

(A.309) hold and

$$\frac{\pi}{R(\underline{s})} < \frac{\omega}{1 - [\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho)]\,(1-\omega)}, \tag{A.333}$$

$$\text{and } \rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \frac{\omega}{1-\omega}(1-k) < 1. \tag{A.334}$$

Then $\overline{\gamma}_F^{NR} \in (0,1)$ and the sufficient replacement threshold in Equation (A.328) is strictly larger than $\overline{\gamma}_F^{NR}$. Hence there is a nonempty interval of γ_F for which full insurance is infeasible without replacement but feasible with replacement by incumbent sellers. Fix any γ_F in this interval.

(A) If only fragile sellers default under the optimal no-replacement contract, replacement is efficient if and only if

$$m^R \leq g_F^{NR}. \tag{A.335}$$

It is strictly efficient if the inequality is strict. If, in addition, Condition (A.322) fails, the no-replacement contract with only fragile-seller defaults strictly dominates that with fragile- and mediocre-seller defaults. Hence $m^R < g_F^{NR}$ implies that replacement strictly dominates both no-replacement contracts.

(B) If fragile and mediocre sellers default under the optimal no-replacement contract, replacement is efficient if and only if

$$m^R - m_{FM}^{NR} \leq g_{FM}^{NR}. \tag{A.336}$$

A sufficient condition for replacement to be strictly efficient is

$$\frac{\omega}{1-\omega}(1-k) \leq (1-\underline{\pi})(1-\rho). \tag{A.337}$$

Proposition A.4 separates two economic reasons for replacement. In Case (A), mediocre sellers remain solvent without replacement and allowing them to default would not be optimal. Replacement nevertheless can be efficient because the second canary margin directs the positions of fragile sellers to safe sellers while mediocre sellers continue to support their original positions. The benefit is the elimination of residual consumption risk; the cost is the opportunity cost of the screening margins. Accordingly, the relevant comparison is m^R versus the risk premium g_F^{NR}.

This first case is nonempty. If Condition (A.322) fails, the F-only contract dominates the $F+M$-default contract independently of the curvature of u. At the same time, $g_F^{NR} > 0$ by strict concavity. Since $m^R = \mathbb{P}(\underline{s})(1-k)\gamma_S\pi$, replacement is strictly efficient for

$k < 1$ sufficiently close to one, provided the replacement contract is resource-compatible. Thus, replacement can be optimal even when the optimal no-replacement contract lets only fragile sellers default and deliberately allowing mediocre sellers to default would reduce welfare.

In Case (B), replacement has an additional benefit: it prevents the inefficient destruction of mediocre sellers' balance sheets. Using Equations (A.326) and (A.318), Condition (A.337) is equivalent to $m^R \leq m^{NR}_{FM}$. Replacement then provides full insurance at a weakly lower markup than the optimal no-replacement contract and is therefore strictly preferred by risk-averse buyers. Together with Conditions (A.322) and (A.325), Equation (A.337) provides a sufficient condition for this second case.

B.6 Proofs

Proof of Proposition A.1. Equation (A.287) implies that fragile sellers cannot make the first-best transfer. Given their default, Equation (A.288) is equivalent to

$$T^{NR}_{F} > \phi_M R(\underline{s}), \tag{A.338}$$

using Equation (A.284). Hence mediocre sellers default as well. Given the joint default of fragile and mediocre sellers, Equation (A.289) is equivalent to

$$T^{NR}_{FM} > R(\underline{s}), \tag{A.339}$$

using Equation (A.286). Thus, safe sellers cannot absorb the remaining loss and full insurance is not resource-compatible. □

Proof of Proposition A.2. The first margin call separates fragile sellers from mediocre and safe sellers if Equation (A.290) holds. Conditional on surviving the first margin call, the replacement margin separates mediocre from safe sellers if Equation (A.292) holds. For a given α, the smallest replacement margin consistent with separation is

$$\beta = \frac{\phi_M - \alpha}{\ell} \tag{A.340}$$

in the limiting sense. Aggregate margin posted is thus

$$(1 - \gamma_F)\alpha + \gamma_F \frac{\phi_M - \alpha}{\ell} = \gamma_M \alpha + \gamma_S \phi_M, \tag{A.341}$$

where we use $\ell = \gamma_F / \gamma_S$. Because $\gamma_M > 0$, aggregate margin is increasing in α. Hence the cost-minimizing first margin is $\alpha^* = \phi_F$, which implies $\beta^* = (\phi_M - \phi_F)/\ell$.

Taking expectations of the CCP budget constraints gives

$$\mathbb{E}[T_B] + \mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma_F \rho \phi_F R(\underline{s}) - \mathbb{P}(\underline{s})\gamma_F C = 0. \tag{A.342}$$

Using the binding participation constraint in Equation (A.297) and the optimal margins in Equation (A.295) yields Equation (A.298). Full insurance implies buyer consumption $\tilde{e} - T_B = \pi - m^R$. Combining this condition with the state-by-state CCP budget constraints yields Equations (A.299)–(A.302).

At the optimal margins, mediocre sellers post ϕ_F on their original position and therefore have resources given by Equation (A.303). Safe sellers post total margin $\phi_F + \ell\beta^* = \phi_M$ and have resources given by Equation (A.304). Since each safe seller makes payments on $1 + \ell$ positions, their resource constraint is Equation (A.306). These two constraints are necessary and sufficient for both surviving seller types to meet their obligations under the full-insurance contract. □

Proof of Proposition A.3. As in the no-replacement benchmark in the main text, we restrict attention to contracts in which sellers survive at $t = 1$. Since $\phi_F = 0$, the interim resource constraint then requires $\alpha = 0$.

Consider first contracts in which fragile sellers do not default at $t = 2$. Their resource constraint then implies $T_S(\underline{s}, \underline{e}) \leq 0$. Since the optimal no-default contract equalizes consumption across all unconstrained states, this constraint binds. Thus, worst-state buyer consumption is zero. The contract is actuarially fair, so buyer consumption in each of the remaining states equals $\pi/[1 - \mathbb{P}(\underline{s}, \underline{e})]$, as stated in Equation (A.314).

Now suppose only fragile sellers default. Their zero resources imply that the contract has no default cost and is actuarially fair. For any transfer $T_S(\underline{s}, \underline{e}) \leq \pi$, mediocre and safe sellers survive. Conditional on this default set, increasing $T_S(\underline{s}, \underline{e})$ transfers consumption from the three non-worst states to the worst state without changing expected consumption. Since full insurance would require a surviving-seller transfer strictly above π for every $\gamma_F > 0$, buyer consumption in the worst state remains below consumption in the other states throughout this region. Strict concavity therefore implies that the optimal transfer is the largest one consistent with mediocre-seller survival, $T_S(\underline{s}, \underline{e}) = \pi$. The CCP budget constraint then gives Equation (A.316), and actuarial fairness together with equal consumption across the other three states gives Equation (A.317). Since $c_F^{NR} = (1 - \gamma_F)\pi > 0$, while both the no-default and F-default contracts have expected consumption π, the latter is a strict mean-preserving contraction of the former. Strict concavity therefore implies that the no-default contract is strictly dominated.

Because seller resources are ordered, the only remaining default sets consistent with survival of safe sellers are $\{F\}$ and $\{F, M\}$.

Now suppose fragile and mediocre sellers default. The markup is the unrecovered fraction of mediocre sellers' resources in the default state, which gives Equation (A.318). Conditional on this default set, the markup is fixed and an increase in $T_S(\underline{s}, \underline{e})$ again reallocates consumption toward the worst state without changing expected consumption. Since $\gamma_F > \overline{\gamma}_F^{NR}$, full insurance cannot be achieved even when safe sellers exhaust their resources. Strict concavity therefore implies that their resource constraint binds, $T_S(\underline{s}, \underline{e}) = R(\underline{s})$. The worst-state CCP budget constraint yields Equation (A.319). Expected buyer consumption equals $\pi - m_{FM}^{NR}$, which together with equal consumption in all other states yields Equation (A.320).

It remains to compare the two candidates. Their worst-state consumption difference is Equation (A.321). If it is weakly negative, the joint-default contract delivers weakly lower consumption in the worst state and, from Equations (A.317) and (A.320), strictly lower consumption in the remaining states. It is therefore dominated. If Equation (A.322) holds, the joint-default contract increases worst-state consumption at the cost of lower consumption in the remaining states, and the exact welfare comparison is Equation (A.323). This proves the characterization.

For completeness, let

$$\Delta_L := c_{FM}^{NR} - c_F^{NR} > 0,$$
$$\Delta_H := \bar{c}_F^{NR} - \bar{c}_{FM}^{NR} = \frac{\mathbb{P}(\underline{s}, \underline{e})}{1 - \mathbb{P}(\underline{s}, \underline{e})} \gamma_S [R(\underline{s}) - \pi].$$

By concavity,

$$u(c_{FM}^{NR}) - u(c_F^{NR}) \geq u'(c_{FM}^{NR}) \Delta_L,$$
$$u(\bar{c}_F^{NR}) - u(\bar{c}_{FM}^{NR}) \leq u'(\bar{c}_{FM}^{NR}) \Delta_H.$$

Hence Equation (A.323) is implied by

$$\mathbb{P}(\underline{s}, \underline{e}) u'(c_{FM}^{NR}) \Delta_L \geq [1 - \mathbb{P}(\underline{s}, \underline{e})] \, u'(\bar{c}_{FM}^{NR}) \Delta_H.$$

Substituting the expressions for Δ_L and Δ_H gives

$$u'(c_{FM}^{NR}) \left(\gamma_S (R(\underline{s}) - \pi) - (1 - \rho) \gamma_M \pi \right) \geq u'(\bar{c}_{FM}^{NR}) \gamma_S (R(\underline{s}) - \pi).$$

Dividing by the positive terms then yields

$$\frac{u'(c_{FM}^{NR})}{u'(\bar{c}_{FM}^{NR})} \geq \frac{\gamma_S(R(\underline{s}) - \pi)}{\gamma_S[R(\underline{s}) - \pi] - (1-\rho)\gamma_M \pi},$$

which is Equation (A.325). □

Proof of Proposition A.4. Condition (A.333) is equivalent to $\overline{\gamma}_F^{NR} > 0$. Under Equation (A.308), fragile default implies mediocre default under a full-insurance contract with late defaults for every $\gamma_F > 0$, so Equation (A.312) is the corresponding full-insurance threshold.

The sufficient replacement condition is Equation (A.328). A direct comparison of its right-hand side with Equation (A.312) shows that the former is larger if and only if

$$\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \frac{\omega}{1-\omega}(1-k) < 1, \tag{A.343}$$

which is Condition (A.334). Hence the interval in which replacement restores full insurance is nonempty.

Fix γ_F in this interval. With replacement, buyers obtain constant consumption $\pi - m^R$ and therefore expected utility $u(\pi - m^R)$.

Suppose first that only fragile sellers default under the optimal no-replacement contract. Because $\phi_F = 0$, this contract is actuarially fair and expected buyer consumption equals π. By the definition of g_F^{NR} in Equation (A.329), its expected utility is $u(\pi - g_F^{NR})$. Since u is strictly increasing, replacement is efficient if and only if $m^R \leq g_F^{NR}$, and strictly efficient if the inequality is strict. If Condition (A.322) fails, Proposition A.3 implies that the F-only contract strictly dominates the $F+M$-default contract. Hence $m^R < g_F^{NR}$ implies that replacement strictly dominates both no-replacement candidates.

Suppose next that fragile and mediocre sellers default under the optimal no-replacement contract. Expected buyer consumption is $\pi - m_{FM}^{NR}$. By the definition of g_{FM}^{NR} in Equation (A.331), expected utility under no replacement is $u(\pi - m_{FM}^{NR} - g_{FM}^{NR})$. Since u is strictly increasing, replacement is efficient if and only if

$$m^R - m_{FM}^{NR} \leq g_{FM}^{NR}. \tag{A.344}$$

Under Equations (A.307)–(A.309), the markup in the $F+M$-default contract is

$$\begin{aligned} m_{FM}^{NR} &= \mathbb{P}(\underline{s}, \underline{e})(1-\rho)\gamma_M \phi_M R(\underline{s}) && \text{(A.345)} \\ &= \mathbb{P}(\underline{s})(1-\underline{\pi})(1-\rho)(1-\omega)(1-\gamma_F)\pi. && \text{(A.346)} \end{aligned}$$

Using Equation (A.326), the inequality $m^R \leq m^{NR}_{FM}$ is equivalent to

$$\frac{\omega}{1-\omega}(1-k) \leq (1-\underline{\pi})(1-\rho), \tag{A.347}$$

which is Condition (A.337). Since replacement provides full insurance while the no-replacement contract provides only partial insurance in the relevant interval, $m^R \leq m^{NR}_{FM}$ implies strictly higher expected utility under replacement. □

C Bilateral Trading

This appendix considers an alternative market structure in which protection buyers trade bilaterally with protection sellers. A buyer is randomly matched with one seller at $t = 0$ and remains exposed to the survival of this seller. Hence, in contrast to central clearing, payments are not pooled across counterparties. The extension allows us to isolate the role of counterparty replacement from the loss-sharing mechanism provided by the CCP.

C.1 Summary of results

The bilateral model delivers three results. First, in the absence of defaults, bilateral trading implements the same allocation as central clearing. The CCP is then superfluous because the contractual payment made by a seller is exactly the payment received by its buyer. If fragile sellers' resources are insufficient to implement the first-best contract, the optimal contract without defaults provides partial insurance and uses no margins, as in Proposition 4.

Second, the absence of central clearing prevents loss sharing across buyers in case of default. If a fragile seller defaults at $t = 2$, the buyer matched with this seller bears the resulting shortfall on her own. The optimal contract therefore equalizes consumption across all states in which the seller survives, while consumption is lower for a buyer whose fragile seller defaults in state $(\underline{s}, \underline{e})$. Margins can improve recovery from a defaulting seller, but impose an opportunity cost on all sellers after a negative signal. In particular, no margin is used when $k \leq \rho$.

Third, replacement remains valuable without central clearing because it reduces the counterparty risk borne directly by buyers. A canary margin can identify fragile sellers at $t = 1$ and allow their buyers to transfer the contract to outsiders. Outsiders insure the remaining endowment risk conditional on the negative signal. In a benchmark with completely impaired fragile sellers, $\phi = 0$, and no replacement cost, $C = 0$, replacement is strictly optimal whenever it is resource-compatible.

C.2 Bilateral contracts

The timing, endowment risk, public signal, seller resources, margins, and outside sellers are as in the main model. We change only the trading arrangement. Each buyer is matched with one seller at $t = 0$, and the derivative contract remains bilateral.

It is useful to distinguish the transfer promised by the contract from the transfer actually received by the buyer. Let

$$\widehat{T}_S(\tilde{s}, \tilde{e}) \tag{A.348}$$

denote the contractual transfer made by the seller. Positive values denote payments by the seller to the buyer. Let

$$T_S(\tilde{s}, \tilde{e}, j), \qquad j \in \{S, F\}, \tag{A.349}$$

denote the actual transfer received by the buyer depending on whether the original seller j is safe or fragile. Buyer consumption is therefore

$$\tilde{e} + T_S(\tilde{s}, \tilde{e}, j). \tag{A.350}$$

If the seller survives, actual and contractual transfers coincide. If the seller defaults, the buyer receives only the amount that can be recovered from the seller or provided by a replacement counterparty.

As in the main model, after a negative signal a safe seller has resources $R(\underline{s})$ and a fragile seller has resources $\phi R(\underline{s})$. A seller of type j that posts margin α and survives until $t = 2$ has resources

$$\bar{t}_{j,2} = \phi_j R(\underline{s}) - (1-k)\alpha R(\underline{s}), \qquad \phi_S = 1, \quad \phi_F = \phi. \tag{A.351}$$

A fragile seller can meet the interim margin call if and only if $\alpha \leq \phi$.

C.3 Contracts without replacement

We first characterize bilateral contracts in which the original seller is not replaced.

No defaults. Suppose sellers never default. Actual and contractual transfers then coincide. The seller participation constraint is

$$0 \leq -\mathbb{E}[\widehat{T}_S] - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}), \tag{A.352}$$

and the fragile seller's resource constraint in state $(\underline{s}, \underline{e})$ is

$$\widehat{T}_S(\underline{s}, \underline{e}) \leq \phi R(\underline{s}) - (1-k)\alpha R(\underline{s}). \tag{A.353}$$

As in the corresponding centrally cleared contract, margins tighten both constraints and are therefore not used.

Proposition A.5 (Bilateral contract without defaults)**.** *The first-best allocation is feasible under bilateral trading without defaults if and only if*

$$\phi R(\underline{s}) \geq \pi. \tag{A.354}$$

If $\phi R(\underline{s}) < \pi$, the optimal contract without defaults uses no margin and provides partial insurance. The fragile seller's resource constraint binds in state $(\underline{s}, \underline{e})$,

$$\widehat{T}_S^{ND}(\underline{s}, \underline{e}) = \phi R(\underline{s}), \tag{A.355}$$

and buyer consumption is constant across all other states and equal to

$$c^{ND} = \frac{\pi - \mathbb{P}(\underline{s}, \underline{e})\phi R(\underline{s})}{1 - \mathbb{P}(\underline{s}, \underline{e})}. \tag{A.356}$$

Thus, for $(\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e})$,

$$\widehat{T}_S^{ND}(\tilde{s}, \tilde{e}) = c^{ND} - \tilde{e}. \tag{A.357}$$

The allocation in Proposition A.5 coincides with the corresponding allocation under central clearing. When there are no defaults, pooling payments through the CCP does not affect risk sharing.

Default at $t = 2$. We next allow fragile sellers to default in state $(\underline{s}, \underline{e})$. Fragile sellers survive the interim date if $\alpha \leq \phi$. If they default at maturity, the transfer recovered by their buyers is

$$r_{F,2}(\alpha) = [\rho\phi + \alpha(k-\rho)]\, R(\underline{s}). \tag{A.358}$$

The seller participation constraint becomes

$$\begin{aligned} 0 = &-\mathbb{E}[\widehat{T}_S] - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) \\ &+ \gamma\mathbb{P}(\underline{s},\underline{e})\left[\widehat{T}_S(\underline{s},\underline{e}) + \big((1-k)\alpha - \phi\big)R(\underline{s})\right]. \end{aligned} \tag{A.359}$$

Conditional on this default pattern and a slack resource constraint for safe sellers, buyer consumption is equalized across all states in which the seller survives. Denote this consumption level by

$$c^D(\alpha) := \frac{\pi - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) + \gamma\mathbb{P}(\underline{s},\underline{e})\big((1-k)\alpha - \phi\big)R(\underline{s})}{1 - \gamma\mathbb{P}(\underline{s},\underline{e})}. \tag{A.360}$$

Accordingly,

$$\widehat{T}_S^D(\tilde{s},\tilde{e}) = c^D(\alpha) - \tilde{e}, \tag{A.361}$$

while a buyer matched with a fragile seller in state $(\underline{s},\underline{e})$ receives $r_{F,2}(\alpha)$.

Proposition A.6 (Bilateral contract with default at $t=2$). *Suppose fragile sellers default at $t=2$ only in state $(\underline{s},\underline{e})$ and safe sellers can make the contractual transfer. The optimal bilateral contract conditional on this default pattern maximizes*

$$\begin{aligned} U^D(\alpha) = &\ [1 - \gamma\mathbb{P}(\underline{s},\underline{e})]\, u(c^D(\alpha)) \\ &+ \gamma\mathbb{P}(\underline{s},\underline{e})u(r_{F,2}(\alpha)) \end{aligned} \tag{A.362}$$

over margin requirements satisfying

$$0 \le \alpha \le \phi, \qquad \phi R(\underline{s}) - (1-k)\alpha R(\underline{s}) < c^D(\alpha) \le R(\underline{s}) - (1-k)\alpha R(\underline{s}). \tag{A.363}$$

At an interior optimum, the margin satisfies

$$\begin{aligned} &\gamma\mathbb{P}(\underline{s},\underline{e})(k-\rho)u'(r_{F,2}(\alpha)) \\ &\quad = [\mathbb{P}(\underline{s}) - \gamma\mathbb{P}(\underline{s},\underline{e})]\,(1-k)u'(c^D(\alpha)). \end{aligned} \tag{A.364}$$

If $k \le \rho$ and $\alpha = 0$ is feasible, the optimal contract uses no margin.

The margin trade-off differs from central clearing because there is no loss sharing. A larger margin improves the payment received by a buyer exposed to a defaulting seller only if $k > \rho$. At the same time, the opportunity cost of margin reduces the resources available in all states in which sellers survive. Hence margins are never useful when the return on the margin account is weakly below the recovery rate on uncollateralized assets.

C.4 Replacement after a negative signal

We now allow a buyer whose seller is fragile to replace this seller at $t = 1$. Following a negative signal, a margin call induces fragile sellers to default if $\alpha > \phi$. Safe sellers survive if $\alpha \leq 1$. Since the margin serves only to identify fragile sellers, the cost-minimizing canary margin is, in the limiting sense used in the main text,

$$\alpha^R = \phi. \tag{A.365}$$

Upon the interim default, the buyer recovers $\rho\phi R(\underline{s})$ from the fragile seller and transfers the position to an outsider. The outsider takes over the contractual transfer $\widehat{T}_S(\underline{s}, \tilde{e})$ in return for a fee p. Competition among risk-neutral outsiders implies

$$p = \mathbb{E}[\widehat{T}_S \mid \underline{s}]. \tag{A.366}$$

Replacement also generates the deadweight cost C. Thus, if the original seller is fragile, the actual transfer after a negative signal is

$$T_S^R(\underline{s}, \tilde{e}, F) = \rho\phi R(\underline{s}) - p - C + \widehat{T}_S^R(\underline{s}, \tilde{e}). \tag{A.367}$$

If the original seller is safe, actual and contractual transfers coincide.

Protection sellers' ex-ante participation constraint is

$$\begin{aligned} 0 = & -\mathbb{E}[\widehat{T}_S] + \mathbb{P}(\underline{s})\gamma \left(\mathbb{E}[\widehat{T}_S \mid \underline{s}] - \phi R(\underline{s})\right) \\ & - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s}). \end{aligned} \tag{A.368}$$

Proposition A.7 (Bilateral contract with replacement)**.** *Suppose fragile sellers default at $t = 1$ and are replaced by outsiders. The optimal canary margin is given by Equation (A.365). The contractual transfer takes the form*

$$\widehat{T}_S^R(\tilde{s}, \tilde{e}) = c_N^R - \tilde{e}, \tag{A.369}$$

where consumption of a buyer whose original seller is not replaced is

$$c_N^R = \frac{\pi - \mathbb{P}(\underline{s})\gamma\left[\underline{\pi} + \phi R(\underline{s})\right] - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\phi R(\underline{s})}{1 - \mathbb{P}(\underline{s})\gamma}. \tag{A.370}$$

A buyer whose fragile seller is replaced after a negative signal receives

$$T_S^R(\underline{s}, \tilde{e}, F) = \underline{\pi} - \tilde{e} + \rho\phi R(\underline{s}) - C, \tag{A.371}$$

and therefore consumes

$$c_F^R = \underline{\pi} + \rho\phi R(\underline{s}) - C. \tag{A.372}$$

Expected buyer utility is

$$\begin{aligned} U^R = &\left[1 - \mathbb{P}(\underline{s})\gamma\right] u(c_N^R) \\ &+ \mathbb{P}(\underline{s})\gamma u\left(\underline{\pi} + \rho\phi R(\underline{s}) - C\right). \end{aligned} \tag{A.373}$$

Replacement is resource-compatible if

$$c_N^R \leq \left[1 - (1-k)\phi\right] R(\underline{s}), \tag{A.374}$$

$$R^O \geq \underline{\pi}. \tag{A.375}$$

Replacement provides full insurance against the endowment realization conditional on the buyer being exposed to a fragile seller after the negative signal: the transfer in Equation (A.371) leaves consumption independent of $\tilde{e}$. However, replacement occurs after the signal has been observed. Outsiders therefore cannot insure the risk associated with the signal itself. This is the bilateral counterpart of the Hirshleifer effect.

C.5 When is bilateral replacement efficient?

We now specialize to the benchmark

$$C = 0, \qquad \phi = 0. \tag{A.376}$$

Fragile sellers then have no resources after a negative signal. Consequently, late default entails no deadweight loss and the canary margin required for replacement has no opportunity cost in the limiting sense.

Under replacement, Equations (A.370) and (A.372) become

$$c_N^R(\gamma) = \frac{\pi - \mathbb{P}(\underline{s})\gamma\underline{\pi}}{1 - \mathbb{P}(\underline{s})\gamma}, \tag{A.377}$$

$$c_F^R = \underline{\pi}. \tag{A.378}$$

Hence replacement preserves expected buyer consumption at π while reducing the consumption risk generated by exposure to fragile sellers.

For comparison, the optimal no-default allocation has buyer consumption

$$0 \quad \text{in state } (\underline{s}, \underline{e}), \qquad \frac{\pi}{1 - \mathbb{P}(\underline{s}, \underline{e})} \quad \text{otherwise.} \tag{A.379}$$

If fragile sellers instead default at $t = 2$, the relaxed optimum that ignores the safe-seller resource constraint has consumption

$$\begin{aligned} 0 &\quad \text{with probability } \gamma\mathbb{P}(\underline{s}, \underline{e}), \\ \frac{\pi}{1 - \gamma\mathbb{P}(\underline{s}, \underline{e})} &\quad \text{otherwise.} \end{aligned} \tag{A.380}$$

If the safe-seller resource constraint binds, attainable utility is weakly lower than under this relaxed allocation.

It is also useful to consider an interim default without replacement. With $\phi = 0$, an arbitrarily small margin can identify fragile sellers at zero opportunity cost in the limiting sense. If their contracts are then terminated rather than replaced, buyers exposed to fragile sellers after $\underline{s}$ simply consume their endowment $\tilde{e}$. The optimal consumption of all buyers whose contracts remain active equals $c_N^R(\gamma)$ in Equation (A.377). Replacement therefore changes only the consumption of buyers exposed to fragile sellers after the negative signal, replacing $\tilde{e} \in \{0, 1\}$ by its conditional expectation $\underline{\pi}$.

Proposition A.8 (Efficiency of replacement under bilateral trading)**.** *Suppose $C = 0$, $\phi = 0$, and $\gamma > 0$. If the replacement contract is resource-compatible,*

$$c_N^R(\gamma) \leq R(\underline{s}), \qquad R^O \geq \underline{\pi}, \tag{A.381}$$

then replacement strictly dominates bilateral contracts that (i) prevent default, (ii) allow fragile sellers to default at $t = 2$, or (iii) induce fragile sellers to default at $t = 1$ and terminate their contracts without replacement. Hence replacement is efficient among these bilateral arrangements.

A sufficient condition for Equation (A.381) to hold for every $\gamma \in (0, 1]$ is

$$R(\underline{s}) \geq \overline{\pi}, \qquad R^O \geq \underline{\pi}. \tag{A.382}$$

Proposition A.8 highlights a difference between bilateral trading and central clearing. Under central clearing, replacement increases aggregate risk-sharing capacity because outsiders reduce the loss-sharing burden on surviving clearing members. This benefit becomes particularly important when fragility is widespread. Under bilateral trading, there is no loss sharing to begin with. Replacement instead benefits the individual buyer who would otherwise remain exposed to a fragile counterparty. In the costless benchmark, this insurance benefit is sufficient to make replacement efficient whenever replacement counterparties have sufficient resources.

The bilateral model also illustrates the timing constraint on replacement particularly clearly. Even when replacement is costless, outsiders arrive only after $\underline{s}$ has been observed. They can therefore insure the remaining endowment risk, but cannot undo the information revealed by the signal. For $\gamma = 1$, buyer consumption under replacement is exactly $\mathbb{E}[\tilde{e} \mid \tilde{s}]$: it equals $\overline{\pi}$ after a positive signal and $\underline{\pi}$ after a negative signal.

C.6 Proofs

Proof of Proposition A.5. The buyer chooses contractual transfers and the margin to maximize expected utility subject to Equations (A.352) and (A.353). Let ξ denote the multiplier on the seller participation constraint and σ that on the fragile seller's resource constraint in state $(\underline{s}, \underline{e})$. The first-order conditions with respect to contractual transfers are

$$\mathbb{P}(\tilde{s}, \tilde{e}) u'\left(\tilde{e} + \widehat{T}_S(\tilde{s}, \tilde{e})\right) - \xi \mathbb{P}(\tilde{s}, \tilde{e}) = 0, \qquad (\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e}), \tag{A.383}$$

$$\mathbb{P}(\underline{s}, \underline{e}) u'\left(\widehat{T}_S(\underline{s}, \underline{e})\right) - \xi \mathbb{P}(\underline{s}, \underline{e}) - \sigma = 0. \tag{A.384}$$

The participation constraint binds, so $\xi > 0$. The derivative of the Lagrangian with respect to the margin is

$$-\xi \mathbb{P}(\underline{s})(1-k)R(\underline{s}) - \sigma(1-k)R(\underline{s}) < 0. \tag{A.385}$$

Hence the optimal contract uses no margin, $\alpha = 0$.

Full insurance. Suppose first that the fragile seller's resource constraint is slack, so $\sigma = 0$. The first-order conditions then imply

$$u'\left(\tilde{e} + \widehat{T}_S(\tilde{s}, \tilde{e})\right) = \xi \qquad \forall(\tilde{s}, \tilde{e}), \tag{A.386}$$

and therefore buyer consumption is constant across states. Since the participation constraint binds and $\alpha = 0$,

$$\mathbb{E}[\widehat{T}_S] = 0, \tag{A.387}$$

so expected buyer consumption is

$$\mathbb{E}[\tilde{e} + \widehat{T}_S] = \pi. \tag{A.388}$$

Full insurance therefore requires consumption π in every state and, in particular,

$$\widehat{T}_S(\underline{s}, \underline{e}) = \pi. \tag{A.389}$$

The fragile seller can make this payment if and only if

$$\pi \leq \phi R(\underline{s}), \tag{A.390}$$

which is Equation (A.354).

Partial insurance. If $\phi R(\underline{s}) < \pi$, the first-best allocation violates the fragile seller's resource constraint. Hence this constraint binds, $\sigma > 0$, and

$$\widehat{T}_S^{ND}(\underline{s}, \underline{e}) = \phi R(\underline{s}), \tag{A.391}$$

which gives Equation (A.355). For all remaining states, the first-order conditions imply

$$u'\left(\tilde{e} + \widehat{T}_S^{ND}(\tilde{s}, \tilde{e})\right) = \xi, \qquad (\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e}), \tag{A.392}$$

so consumption is equalized at a common level c^{ND} outside the worst state. Moreover, the first-order condition in the worst state gives

$$u'(\phi R(\underline{s})) = \xi + \frac{\sigma}{\mathbb{P}(\underline{s}, \underline{e})} > \xi, \tag{A.393}$$

confirming that consumption is strictly lower in that state.

Using again that the participation constraint binds and $\alpha = 0$,

$$0 = \mathbb{E}[\widehat{T}_S] \quad \Longleftrightarrow \quad \mathbb{E}[\tilde{e} + \widehat{T}_S] = \pi. \tag{A.394}$$

Therefore,

$$\mathbb{P}(\underline{s}, \underline{e})\phi R(\underline{s}) + [1 - \mathbb{P}(\underline{s}, \underline{e})]\, c^{ND} = \pi, \tag{A.395}$$

which implies

$$c^{ND} = \frac{\pi - \mathbb{P}(\underline{s}, \underline{e})\phi R(\underline{s})}{1 - \mathbb{P}(\underline{s}, \underline{e})}. \tag{A.396}$$

Thus, in every other state,

$$\widehat{T}_S^{ND}(\tilde{s}, \tilde{e}) = c^{ND} - \tilde{e}, \tag{A.397}$$

which gives Equations (A.356) and (A.357). □

Proof of Proposition A.6. In the default event $(\underline{s}, \underline{e}, F)$, a fragile seller does not make the contractual transfer and loses its assets. Relative to the no-default participation constraint, the correction to the seller's payoff is therefore

$$\widehat{T}_S(\underline{s}, \underline{e}) + \big((1 - k)\alpha - \phi\big) R(\underline{s}). \tag{A.398}$$

Since the default event has probability $\gamma \mathbb{P}(\underline{s}, \underline{e})$, the seller participation constraint is Equation (A.359).

Risk sharing. Let ξ denote the multiplier on the seller participation constraint and suppose that the safe seller's resource constraint is slack. For $(\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e})$, all matched sellers make the contractual transfer, and the first-order condition is

$$\mathbb{P}(\tilde{s}, \tilde{e}) u'\Big(\tilde{e} + \widehat{T}_S(\tilde{s}, \tilde{e})\Big) - \xi \mathbb{P}(\tilde{s}, \tilde{e}) = 0. \tag{A.399}$$

In state $(\underline{s}, \underline{e})$, only the mass $1 - \gamma$ of safe sellers makes the contractual transfer. Hence the corresponding first-order condition is

$$(1 - \gamma)\mathbb{P}(\underline{s}, \underline{e}) u'\Big(\widehat{T}_S(\underline{s}, \underline{e})\Big) - \xi(1 - \gamma)\mathbb{P}(\underline{s}, \underline{e}) = 0. \tag{A.400}$$

Thus, buyer consumption is equalized across all events in which the seller survives. Denoting this common consumption level by $c^D(\alpha)$ gives

$$\widehat{T}_S^D(\tilde{s}, \tilde{e}) = c^D(\alpha) - \tilde{e}. \tag{A.401}$$

Substituting this transfer into the binding participation constraint yields

$$\begin{aligned} 0 = & -\left(c^D(\alpha) - \pi\right) - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) \\ & + \gamma \mathbb{P}(\underline{s}, \underline{e}) \left[c^D(\alpha) + \left((1-k)\alpha - \phi\right) R(\underline{s})\right] \end{aligned} \tag{A.402}$$

$$\begin{aligned} = & \ \pi - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) + \gamma \mathbb{P}(\underline{s}, \underline{e})\left((1-k)\alpha - \phi\right) R(\underline{s}) \\ & - \left[1 - \gamma \mathbb{P}(\underline{s}, \underline{e})\right] c^D(\alpha). \end{aligned} \tag{A.403}$$

Solving for $c^D(\alpha)$ gives Equation (A.360). In the default event, buyer consumption equals the recovery

$$r_{F,2}(\alpha) = \left[\rho\phi + \alpha(k - \rho)\right] R(\underline{s}), \tag{A.404}$$

so expected utility is Equation (A.362).

The restrictions in Equation (A.363) follow directly from the assumed default pattern. First, $\alpha \leq \phi$ ensures that fragile sellers survive the interim margin call. Second, fragile sellers default at $t = 2$ in $(\underline{s}, \underline{e})$ if

$$c^D(\alpha) > \phi R(\underline{s}) - (1-k)\alpha R(\underline{s}), \tag{A.405}$$

while safe sellers survive if

$$c^D(\alpha) \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}). \tag{A.406}$$

Optimal margin. Differentiating Equations (A.360) and (A.358) gives

$$\frac{dc^D}{d\alpha} = -\frac{\left[\mathbb{P}(\underline{s}) - \gamma \mathbb{P}(\underline{s}, \underline{e})\right](1-k) R(\underline{s})}{1 - \gamma \mathbb{P}(\underline{s}, \underline{e})}, \tag{A.407}$$

$$\frac{dr_{F,2}}{d\alpha} = (k - \rho) R(\underline{s}). \tag{A.408}$$

At an interior optimum,

$$0 = \frac{dU^D}{d\alpha} \tag{A.409}$$

$$= [1 - \gamma \mathbb{P}(\underline{s}, \underline{e})]\, u'(c^D(\alpha)) \frac{dc^D}{d\alpha} + \gamma \mathbb{P}(\underline{s}, \underline{e}) u'(r_{F,2}(\alpha)) \frac{dr_{F,2}}{d\alpha} \tag{A.410}$$

$$= -\left[\mathbb{P}(\underline{s}) - \gamma \mathbb{P}(\underline{s}, \underline{e})\right](1-k) R(\underline{s}) u'(c^D(\alpha)) + \gamma \mathbb{P}(\underline{s}, \underline{e})(k - \rho) R(\underline{s}) u'(r_{F,2}(\alpha)). \tag{A.411}$$

Dividing by $R(\underline{s}) > 0$ and rearranging gives Equation (A.364). If $k \leq \rho$, then $dr_{F,2}/d\alpha \leq 0$, while $dc^D/d\alpha < 0$. Hence $dU^D/d\alpha < 0$ throughout the feasible set, and $\alpha = 0$ is optimal whenever it is feasible. □

Proof of Proposition A.7. Competition among risk-neutral outsiders implies that their participation constraint binds, so

$$p = \mathbb{E}[\widehat{T}_S \mid \underline{s}], \tag{A.412}$$

which is Equation (A.366). Let ξ denote the multiplier on the original sellers' participation constraint. For the risk-sharing part of the proof, define buyer consumption when the original seller is not replaced by

$$c_N(\tilde{s}, \tilde{e}) := \tilde{e} + \widehat{T}_S(\tilde{s}, \tilde{e}), \tag{A.413}$$

and, after replacement following $\underline{s}$, by

$$c_F(\tilde{e}) := \tilde{e} + \rho\phi R(\underline{s}) - C + \widehat{T}_S(\underline{s}, \tilde{e}) - \mathbb{E}[\widehat{T}_S \mid \underline{s}]. \tag{A.414}$$

Risk sharing. After a positive signal, no seller is replaced. The first-order conditions with respect to $\widehat{T}_S(\bar{s}, \tilde{e})$ are therefore

$$\mathbb{P}(\bar{s}, \tilde{e})\left[u'(c_N(\bar{s}, \tilde{e})) - \xi\right] = 0, \tag{A.415}$$

which implies

$$u'(c_N(\bar{s}, \tilde{e})) = \xi. \tag{A.416}$$

After a negative signal, changing $\widehat{T}_S(\underline{s}, \tilde{e})$ changes the outsider fee p one-for-one with its conditional probability. The first-order condition is therefore

$$\begin{aligned}0 = (1-\gamma)\left[u'(c_N(\underline{s},\tilde{e})) - \xi\right] \\ + \gamma\left[u'(c_F(\tilde{e})) - \mathbb{E}\left[u'(c_F(\tilde{e})) \mid \underline{s}\right]\right].\end{aligned} \tag{A.417}$$

Taking the difference of this condition across the two endowment realizations gives

$$\begin{aligned}0 = (1-\gamma)\left[u'(c_N(\underline{s},\bar{e})) - u'(c_N(\underline{s},\underline{e}))\right] \\ + \gamma\left[u'(c_F(\bar{e})) - u'(c_F(\underline{e}))\right].\end{aligned} \tag{A.418}$$

Moreover,

$$c_F(\bar{e}) - c_F(\underline{e}) = c_N(\underline{s},\bar{e}) - c_N(\underline{s},\underline{e}). \tag{A.419}$$

Since u is strictly concave, the two differences in marginal utility have the same sign unless the corresponding consumption difference is zero. The first-order condition can therefore hold only if

$$c_N(\underline{s},\bar{e}) = c_N(\underline{s},\underline{e}) \quad \text{and} \quad c_F(\bar{e}) = c_F(\underline{e}). \tag{A.420}$$

For $\gamma < 1$, substituting this result back into the first-order condition after $\underline{s}$ yields

$$u'(c_N(\underline{s},\tilde{e})) = \xi, \tag{A.421}$$

so non-replaced buyer consumption is equalized across all states. Denote this common level by c_N^R. Thus,

$$\widehat{T}_S^R(\tilde{s},\tilde{e}) = c_N^R - \tilde{e}, \tag{A.422}$$

which gives Equation (A.369). Conditional on $\underline{s}$,

$$p = \mathbb{E}[\widehat{T}_S^R \mid \underline{s}] = c_N^R - \underline{\pi}. \tag{A.423}$$

Substituting into Equation (A.367) gives

$$\begin{aligned}T_S^R(\underline{s},\tilde{e},F) &= \rho\phi R(\underline{s}) - \left(c_N^R - \underline{\pi}\right) - C + c_N^R - \tilde{e} && \text{(A.424)}\\ &= \underline{\pi} - \tilde{e} + \rho\phi R(\underline{s}) - C, && \text{(A.425)}\end{aligned}$$

which proves Equation (A.371) and implies

$$c_F^R = \underline{\pi} + \rho\phi R(\underline{s}) - C. \tag{A.426}$$

Optimal margin. Fragile sellers default at the interim date when $\alpha > \phi$, whereas safe sellers survive for $\alpha \leq 1$. Conditional on replacement, fragile sellers default before posting margin, so increasing α above the screening threshold does not increase recovery from them. It only weakly raises the opportunity cost borne by safe sellers and tightens their resource constraint. In particular, away from the screening threshold, the direct contribution of the participation constraint to the derivative of the Lagrangian is

$$-\xi\mathbb{P}(\underline{s})(1-\gamma)(1-k)R(\underline{s}) \leq 0, \tag{A.427}$$

with a strict inequality whenever $\gamma < 1$. Hence the cost-minimizing margin is the smallest one that induces fragile sellers to default, $\alpha^R = \phi$ in the limiting sense, as in Equation (A.365).

Transfers and resources. Using

$$\mathbb{E}[\widehat{T}_S] = c_N^R - \pi, \tag{A.428}$$

$$\mathbb{E}[\widehat{T}_S \mid \underline{s}] = c_N^R - \underline{\pi}, \tag{A.429}$$

and $\alpha^R = \phi$ in the binding participation constraint (A.368) gives

$$\begin{aligned} 0 = & -\left(c_N^R - \pi\right) + \mathbb{P}(\underline{s})\gamma\left(c_N^R - \underline{\pi} - \phi R(\underline{s})\right) \\ & - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\phi R(\underline{s}) \end{aligned} \tag{A.430}$$

$$\begin{aligned} = & \ \pi - \mathbb{P}(\underline{s})\gamma\left[\underline{\pi} + \phi R(\underline{s})\right] - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\phi R(\underline{s}) \\ & - \left[1 - \mathbb{P}(\underline{s})\gamma\right] c_N^R. \end{aligned} \tag{A.431}$$

Solving for c_N^R gives Equation (A.370). Together with the two consumption levels derived above, this yields Equation (A.373).

In state $(\underline{s}, \underline{e})$, a safe seller's contractual payment is

$$\widehat{T}_S^R(\underline{s}, \underline{e}) = c_N^R, \tag{A.432}$$

while its resources after posting margin are $[1-(1-k)\phi]R(\underline{s})$. Hence safe sellers are resource-compatible if and only if Equation (A.374) holds. Finally, the largest net payment made by an outsider is

$$\widehat{T}^R_S(\underline{s},\underline{e}) - p = c^R_N - \left(c^R_N - \underline{\pi}\right) \tag{A.433}$$
$$= \underline{\pi}, \tag{A.434}$$

which gives Equation (A.375). □

Proof of Proposition A.8. Under $C = \phi = 0$, replacement is costless. Using Equation (A.377), expected buyer consumption under replacement is

$$\begin{aligned}&[1 - \mathbb{P}(\underline{s})\gamma]\, c^R_N(\gamma) + \mathbb{P}(\underline{s})\gamma\underline{\pi} \\ &\quad = \pi - \mathbb{P}(\underline{s})\gamma\underline{\pi} + \mathbb{P}(\underline{s})\gamma\underline{\pi} = \pi.\end{aligned} \tag{A.435}$$

The replacement allocation has two consumption levels: $\underline{\pi}$ for buyers whose fragile seller is replaced after $\underline{s}$, and $c^R_N(\gamma)$ for all other buyers. Moreover,

$$c^R_N(\gamma) - \pi = \frac{\pi - \mathbb{P}(\underline{s})\gamma\underline{\pi} - \pi\,[1 - \mathbb{P}(\underline{s})\gamma]}{1 - \mathbb{P}(\underline{s})\gamma} \tag{A.436}$$
$$= \frac{\mathbb{P}(\underline{s})\gamma(\pi - \underline{\pi})}{1 - \mathbb{P}(\underline{s})\gamma} > 0, \tag{A.437}$$

for every $\gamma > 0$.

Default at $t = 2$. For $\phi = 0$, the recovery from a fragile seller that defaults at maturity is zero when $\alpha = 0$. Ignoring the safe-seller resource constraint, the first-order conditions in the proof of Proposition A.6 equalize consumption in every non-default event. Since expected buyer consumption remains π, denoting this consumption level by c gives

$$[1 - \gamma\mathbb{P}(\underline{s},\underline{e})]\, c = \pi, \tag{A.438}$$

so

$$c = \frac{\pi}{1 - \gamma\mathbb{P}(\underline{s},\underline{e})}. \tag{A.439}$$

This is the relaxed late-default allocation in Equation (A.380).

Its support is therefore

$$\left\{0, \frac{\pi}{1 - \gamma\mathbb{P}(\underline{s},\underline{e})}\right\}. \tag{A.440}$$

The support of the replacement allocation lies strictly inside this interval. The lower endpoint satisfies $0 < \underline{\pi}$. For the upper endpoint,

$$\begin{aligned} &\frac{\pi}{1-\gamma \mathbb{P}(\underline{s},\underline{e})} - c_N^R(\gamma) \\ &\quad = \frac{\mathbb{P}(\underline{s})\gamma\underline{\pi}\,[1-\pi-\gamma\mathbb{P}(\underline{s},\underline{e})]}{[1-\mathbb{P}(\underline{s})\gamma][1-\gamma\mathbb{P}(\underline{s},\underline{e})]} > 0, \end{aligned} \tag{A.441}$$

where the inequality follows from

$$\gamma\mathbb{P}(\underline{s},\underline{e}) = \gamma\lambda(1-\pi) < 1-\pi. \tag{A.442}$$

Both allocations have mean π. The two-point late-default distribution places mass on the endpoints of an interval that strictly contains the support of the replacement distribution and is therefore a mean-preserving spread of the replacement allocation. Strict concavity of u implies that replacement yields strictly higher expected utility. If the safe-seller resource constraint binds in the late-default problem, attainable utility is weakly lower than in this relaxed problem, so the result continues to hold.

No default. Under $\phi = 0$, Proposition A.5 gives consumption zero in state $(\underline{s},\underline{e})$ and a common consumption level c in all other states. Since mean consumption equals π,

$$[1-\mathbb{P}(\underline{s},\underline{e})]\,c = \pi, \tag{A.443}$$

which yields Equation (A.379). Moreover,

$$\frac{\pi}{1-\mathbb{P}(\underline{s},\underline{e})} \geq \frac{\pi}{1-\gamma\mathbb{P}(\underline{s},\underline{e})} > c_N^R(\gamma), \tag{A.444}$$

while the lower consumption level remains zero. Hence the support of the replacement allocation is also strictly contained in the support of the no-default allocation. Since both have mean π, the same mean-preserving-spread argument implies that replacement strictly dominates the no-default allocation.

Interim default without replacement. Termination and replacement generate the same consumption for all buyers matched with surviving sellers. Conditional on being matched with a fragile seller after $\underline{s}$, termination leaves buyer consumption equal to $\tilde{e}$, whereas replacement replaces this random consumption by its conditional expectation $\underline{\pi}$. Since $\underline{\pi} \in (0,1)$ and u is strictly concave, replacement strictly dominates interim termination.

Resource compatibility. Finally,

$$\frac{dc_N^R(\gamma)}{d\gamma} = \frac{\mathbb{P}(\underline{s})(\pi - \underline{\pi})}{[1 - \mathbb{P}(\underline{s})\gamma]^2} > 0. \tag{A.445}$$

Hence $c_N^R(\gamma)$ is increasing in γ. At $\gamma = 1$,

$$c_N^R(1) = \frac{\pi - \mathbb{P}(\underline{s})\underline{\pi}}{1 - \mathbb{P}(\underline{s})} \tag{A.446}$$

$$= \overline{\pi}, \tag{A.447}$$

where the second equality follows from the law of iterated expectations,

$$\pi = \mathbb{P}(\overline{s})\overline{\pi} + \mathbb{P}(\underline{s})\underline{\pi}. \tag{A.448}$$

Therefore, $R(\underline{s}) \geq \overline{\pi}$ implies the safe-seller resource constraint for every $\gamma \in (0, 1]$. Together with $R^O \geq \underline{\pi}$, this proves the sufficient condition in Equation (A.382). $\square$